Modelling micro/nano- field effect electron devices

MICRO-623
EDOC lecture

Jean-Michel Sallese
EPFL
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Why Analytical and Compact Modelling.

Aim of the course:

Propose simple while accurate analytical models for a variety of field-effect semiconductor devices.

Ideally:

These models must be physics based.

They can tolerate some approximations that must be justified and validated.

They can be implemented in electrical simulators to simulate IC's (Pspice - https://www.pspice.com/…).

What about Numerical simulations ?

++: They are very important to predict with many details the devices characteristics.

- -: Can hardly give a synthetic understanding of the devices behavior.

- -: Cannot be used for hand calculations or IC's simulations.

They are required to validate the analytical models (in addition to experiments).

Example: Synopsys - https://www.synopsys.com/manufacturing/tcad.html

Sylvaco - https://silvaco.com/tcad/

OUTLINE

I- Alternative modeling of the bulk MOSFET.
- Recalling basics of MOSFETs
- Another way to model MOSFETs
- Short Channel effects in MOS transistors.

II- Modelling the Double Gate FETs.
- The exact solution
- The charge based Model of the DG FET
- Quantum Confinement in DG FET
- Including doping in DG FETs

III- The Gate All around FET

IV- Concept of equivalent parameters in arbitrary FETs.
- The equivalent channel thickness
- The equivalent gate capacitance

V- The Junctionless Field Effect transistor.
- The simplest FET
- Pros and Cons of JL FETs
- Modelling the double gate JL FET
- Asymetric operation in DG JL FETs
- The Nanowire FET

VI- Ballistic transport in nanoscale transistors.
- What is ballistic transport ?
- The virtual source and fluxes
- Is the ballistic FET a vacuum tube ?
- Ultimate contact resistance
- The 'molecular' FET

VII- Modeling the High Electron Mobility FETs (HEMT).
- The energy band shape in AlGaAs HEMT heterostructures
- The concept of charge linearization in HEMT
- The III-Nitride AlGaN HEMT

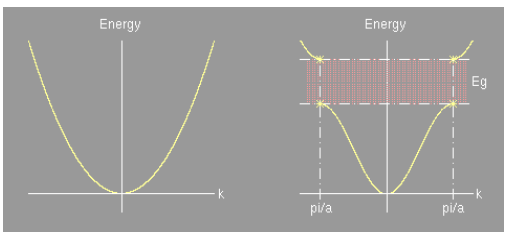
VIII- Modeling biosensor Nanowires FET and Bio-Sensors
- Basics of solid-electrolyte interaction
- Modeling ISFET Nanowires
- Simulations of ISFET Nanowires

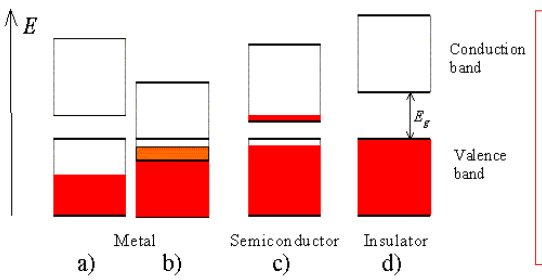
IX- Modeling negative capacitance in multigate FETs
(not sure yet).

Optionnal :
Recalling basics of Semiconductor Physics

Basics of SC physics – Bands

- A band = splitting of one or more atomic energy levels.
- Number of states in a band = twice the number of atoms.





Depending on [valence electrons]:

- Partially filled bands(V or C): conductors
- Completely filled bands:
  - If $E_G/kT >> 1$, electrons cannot gain extra energy= insulators.
  - If $E_G/kT \sim 1$, electrons can gain extra energy= semiconductors.

Basics of SC physics – Bands

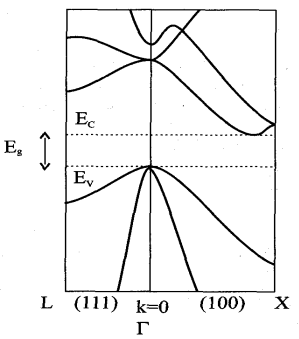
• Wave function continuity at the crystal limits imposes discrete values for **k:**

$$k = n \cdot \left[2 \cdot \pi / N_{atoms} \cdot P \right]$$

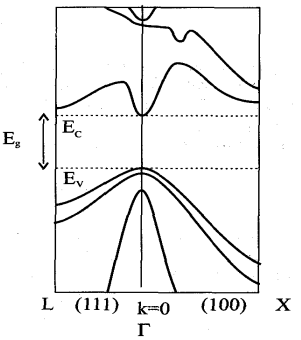
P: crystal periodicity, n: integer

• This limits the representation in the `k space` from -π/*P* to π/*P* : The Brillouin zone

Silicon:
Indirect band gap



Gallium Arsenide:
Direct band gap



• Energy versus the wave vector is represented in the first Brillouin zone of the reciprocal lattice.

L and **X** represent different directions in the cristal.

Carrier velocity at the state 'k'

$$V_k = \frac{1}{\hbar} \cdot \frac{dE}{dk}$$

Basics of SC physics – Effective mass

- How can we `recover` the simple 'free electron' description ?
- Noting that electrical and optical ('visible' range) properties of a semiconductor mainly depend on conduction and valence band extremas, we can develop the energy dependence *E(k)* up to the second order in *k* around the extrema k_0.
- This will define an 'effective mass' m^* which is related to the curvature of the *E(k)* diagram.

  The effective mass concept holds both for electrons and holes.

$$E(k) = E(k_0) + \frac{\hbar^2 \cdot (k - k_0)^2}{2 \cdot m^*} \qquad \frac{1}{m^*} = \frac{1}{\hbar^2} \cdot \frac{d^2 E}{dk^2}$$

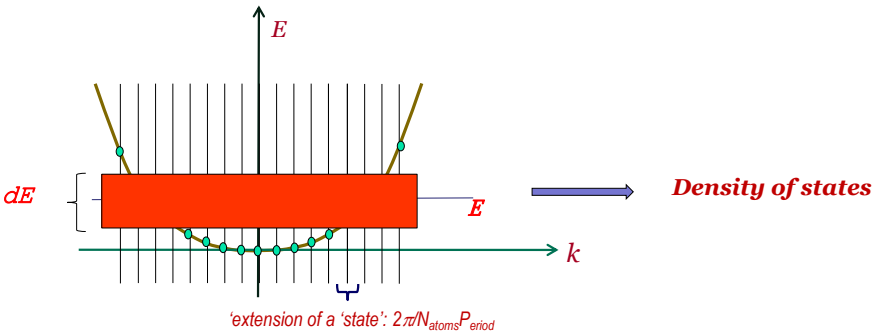
- Energies are measured with respect to the extrema.

$$E_e = E_C + \frac{\hbar^2 \cdot k^2}{2 \cdot m_e^*} \qquad E_h = E_V + \frac{\hbar^2 \cdot k^2}{2 \cdot m_h^*}$$

- The electronic properties of a cristal are implicitely hidden in the effective mass concept.
- In a periodic structure, electrons and holes can seem heavier or lighter than electrons in free space.

Basics of SC physics – Density of States

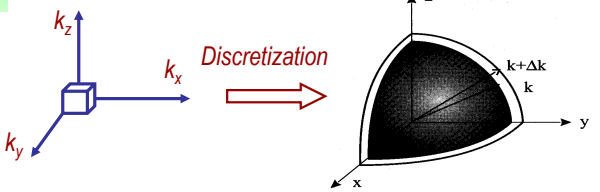
•Discretization of k vector in reciprocal space



• Discrete values of the wave vector k will induce quantization of energies:

- In 1D, a 'k' state occupies a 'length' of: $2\pi/N_{atoms}P_{eriod}$, or $2\pi/L$
- In 3D, a 'k' state occupies a 'volume' of: $8\pi^3/V_{olume}$

Basics of SC physics – Density of States



Nb of states between k and k+Δk:

$$\frac{V}{8\cdot\pi^3}\cdot\left(4\pi\cdot k^2\cdot dk\right)$$

Energy dependent Density of states:

and we have: $E(k)=\dfrac{\hbar^2\cdot k^2}{2\cdot m^*}$ ..... + spin(×2)

$$N_{3D}(E)=\frac{Vol}{2\cdot\pi^2}\cdot\left(\frac{2\cdot m^*}{\hbar^2}\right)^{3/2}\cdot\sqrt{E}$$

In 2D: $N_{2D}(E)=\dfrac{Surf\cdot m^*}{\pi\cdot\hbar^2}$

In 1D: $N_{1D}(E)=\dfrac{Length\cdot\sqrt{m^*}}{\pi\cdot\hbar}\cdot\dfrac{1}{\sqrt{E}}$

(these apply for each confined state. The energy E is then measured wrt the confined level)

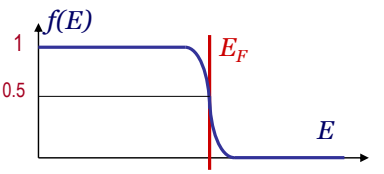
- Example: the density of states at 100 meV above the conduction band in silicon (per m^3 of silicon is) ~ 2 10^{27} states / eV
- In a cube of crystal of 100μm sides, for a 10 meV energy 'window,' , there will be 2 10^{13} states that can be occupied.

Basics of SC physics – Statistics

- How electrons will be distributed on the available states ?

- The Fermi-Dirac statistics gives the probability f(E) that an electron occupies a state of energy E depends on the temperature and on the Fermi Energy E_F:

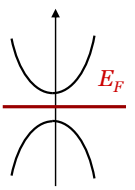
$$f_e(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k \cdot T}\right)}$$



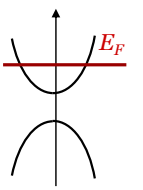
- If the electron density is low enough, i.e. $E\text{-}E_F >> kT$, the distribution reverts to the Bolzmann statistics :

$$f_e(E) \approx \exp\left(\frac{E_F - E}{k \cdot T}\right)$$

$E_C\text{-}E_F >> kT$
Semiconductor
non-degenerated



$E_C\text{-}E_F << kT$
Semiconductor
degenerated
(N type in this case)



Basics of SC physics – Statistics

- Concerning holes, we obtain a similar relation considering that:

Prob (hole @ energy E) = 1 – Prob (electron @ energy E)

$$f_h(E) = \frac{1}{1 + \exp\left(\frac{E_F - E}{k \cdot T}\right)}$$

Degenerate

'lightly doped'

$$f_h(E) \approx \exp\left(\frac{E - E_F}{k \cdot T}\right)$$

Non-degenerate

Basics of SC physics – Density of states

• Given the density of state and the probability that these are occupied allows in evaluating the total electron and hole densities. Electron density in the conduction band and hole density in the valence band are given by:

$$n = \int_0^{\infty} N_c(E) \cdot f_e(E) \cdot dE \qquad p = \int_{-\infty}^{0} N_v(E) \cdot f_h(E) \cdot dE$$

• In case of a non-degenerate semiconductor, we obtain:

$$n \approx N_C \cdot \exp\left(\frac{E_F - E_C}{k \cdot T}\right) \qquad p \approx N_V \cdot \exp\left(\frac{E_V - E_F}{k \cdot T}\right)$$

*Where N_C and N_V represent the **Effective Density of States** for conduction and valence bands:*

$$N_{C,V} = 2 \cdot \left(\frac{2 \cdot \pi \cdot m_{e,h} \cdot k \cdot T}{h^2}\right)^{3/2}$$

• For a fixed T°, the product *n.p* is invariant :

$$n \cdot p = n_i^2 = N_c \cdot N_v \cdot \exp\left(-\frac{E_g}{kT}\right)$$

n_i = intrinsic carrier density

Basics of SC physics – Donors & acceptors

• Excess of default of carriers can be introduced by 'impurities' that have more (donor) or less (acceptor) valence electrons than the semiconductor atoms.

• In silicon, phosphorus gives an extra electron whereas boron generates a lack of electron, or equivalently bring an extra hole.

• Assuming that each donor/acceptor generates a free electron/hole, we have:

$$N_D \approx N_C \cdot \exp\left(\frac{E_F - E_C}{k \cdot T}\right)$$ *For N type doping*

$$N_A \approx N_V \cdot \exp\left(\frac{E_V - E_F}{k \cdot T}\right)$$ *For P type doping*

This uniquely defines the Fermi energy once the doping density is known.

Basics of SC physics – Intrinsic Fermi energy

• The intrinsic energy E_{Fi} corresponds to the Fermi energy that satisfies $n=p=n_i$:

$$n_i = N_C \cdot \exp\left(\frac{E_{Fi} - E_C}{k \cdot T}\right) = N_V \cdot \exp\left(\frac{E_V - E_{Fi}}{k \cdot T}\right) \Rightarrow E_{Fi} = \left(\frac{E_C + E_V}{2}\right) + \frac{kT}{2} \cdot Ln\left(\frac{N_V}{N_C}\right)$$

• Electrons and holes densities take a very simple form relaing on n_i and E_{Fi}:

$$n = n_i \cdot \exp\left(\frac{E_F - E_{Fi}}{k \cdot T}\right) \qquad p = n_i \cdot \exp\left(\frac{E_{Fi} - E_F}{k \cdot T}\right)$$

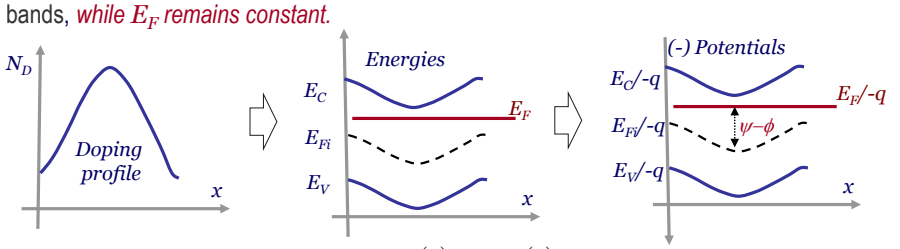
• On the other hand, introducing doping impurities will affect electron and hole densities; and so it will change the Fermi energy. In non-degenerate semiconductor, we have:

$$E_F \approx E_{Fi} + k \cdot T \cdot Ln\left(\frac{N_D}{n_i}\right)$$ *for N type (ex: Phosphorous in Si)*

$$E_F \approx E_{Fi} - k \cdot T \cdot Ln\left(\frac{N_A}{n_i}\right)$$ *for P type (ex: Boron in Si)*

Basics of SC physics – Potentials

• Under equilibrium, non-uniform doping induces bending of conduction and valence bands, *while E_F remains constant.*



Defining the 'band bending' potential as: $\psi(x) = -E_{Fi}(x)/q$

and the Fermi level potential as: $\phi(x) = -E_F(x)/q$

• Electron and hole densities are then given by:

$$n(x) = n_i \cdot \exp\left(\frac{\psi(x) - \phi_n(x)}{U_T}\right) \qquad p(x) = n_i \cdot \exp\left(\frac{\phi_p(x) - \psi(x)}{U_T}\right)$$

where U_T is the thermodynamic potential (25 meV @ RT): $U_T = k \cdot T/q$

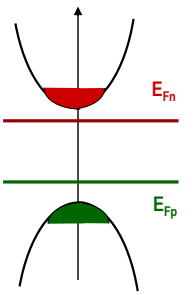
Basics of SC physics – Pseudo Fermi energy

- In order to represent a **non equilibrium** situation (no external potential V), Fermi levels can have different values for electrons and holes at the same coordinate.
- This occurs when electrons and holes exceed their equilibrium densities subsequent to an external exitation, such as light absorption.
- In this case, we can define an electron E_{Fn} and hole E_{Fp} Fermi levels and corresponding potentials such that:

$$n(x) = n_i \cdot \exp\left(\frac{\psi(x) - \phi_n(x)}{U_T}\right)$$

$$p(x) = n_i \cdot \exp\left(\frac{\phi_p(x) - \psi(x)}{U_T}\right)$$

$$n \cdot p = n_i^2 \cdot \exp\left(\frac{\phi_p - \phi_n}{U_T}\right)$$



*(as we will see, applying an external potential **V** will also modify the Fermi level)*

Basics of SC physics – Drift-Diffusion transport

• Drift current due to an electric field:

$$\begin{cases} \vec{J}_{e\_drift}(\vec{r}) = -q \cdot n(\vec{r}) \cdot \mu_e \cdot \vec{E}(\vec{r}) \\ \vec{J}_{h\_drift}(\vec{r}) = -q \cdot p(\vec{r}) \cdot \mu_h \cdot \vec{E}(\vec{r}) \end{cases}$$

Where $q = -1.6\ 10^{-19}\ C$

• Drift current due to a gradient:

$$\begin{cases} \vec{J}_{e\_diff}(\vec{r}) = -q \cdot D_e \cdot \vec{\nabla} n(\vec{r}) \\ \vec{J}_{h\_diff}(\vec{r}) = q \cdot D_h \cdot \vec{\nabla} p(\vec{r}) \end{cases}$$

• The current density is then the sum of the two components:

$$\vec{J}_e(\vec{r}) = \vec{J}_{e\_drift}(\vec{r}) + \vec{J}_{e\_diff}(\vec{r}) \qquad \vec{J}_h(\vec{r}) = \vec{J}_{h\_drift}(\vec{r}) + \vec{J}_{h\_diff}(\vec{r})$$

• The total current density is the sum of electron and hole current densities:

$$\vec{J}_{Total}(\vec{r}) = \vec{J}_e(\vec{r}) + \vec{J}_h(\vec{r})$$

Note that other approaches exist. However, DD is both simple and accurate enough.

Basics of SC physics – Drift-Diffusion transport

• Assuming Boltzmann statistics and noting that : $E = \frac{-d\psi}{dx}$ *and* $D = \mu \cdot U_T$

we obtain: $J_e = q \cdot \mu \cdot \left(\quad + \quad \right)$ *(drift – diffusion)*

In addition: $\frac{dn}{dx} = \frac{n}{U_T} \cdot \left(\frac{d\psi}{dx} - \frac{d\phi_n}{dx} \right)$

• The total current, including drift and diffusion, is then given by:

$$J_e = -q \cdot \mu \cdot n \cdot \frac{d\phi_n}{dx}$$

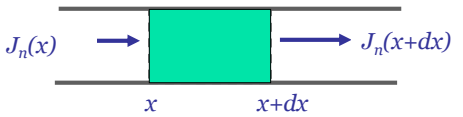
• The same expression is obtained for hole current:

$$J_h = -q \cdot \mu \cdot p \cdot \frac{d\phi_p}{dx}$$

Interestingly, if Fermi potentials are constant, there will be not net current.

Basics of SC physics – The continuity equation

• For a given volume, incoming and outcoming fluxes must be compensated by Generation-Recombination processes.



$$\frac{dn}{dt}\cdot dx = \left[\frac{J_n(x+dx)}{-q} - \frac{J_n(x)}{-q}\right] + (G_n - R_n)\cdot dx$$

$$J_n(x+dx) = J_n(x) + \frac{dJ_n(x)}{dx}\cdot dx$$

$$\Rightarrow \quad \frac{dn}{dt} = -\frac{1}{q}\cdot\frac{dJ_n(x)}{dx} + (G_n - R_n)\cdot dx$$

Derivating 'Drift-Diffusion' gives:

$$\frac{dJ_n(x)}{dx} = -q\cdot\mu\cdot E\cdot\frac{dn}{dx} - q\cdot\mu\cdot n\cdot\frac{dE}{dx} - q\cdot D_n\cdot\frac{d^2n}{dx^2}$$

The continuity equation derived from Drift-Diffusion is given by:

$$\frac{dn}{dt} = \mu_n\cdot n\cdot\frac{dE}{dx} + \mu_n\cdot E\cdot\frac{dn}{dx} + D_n\cdot\frac{d^2n}{dx^2} + G_n - R_n$$

$$\frac{dp}{dt} = -\mu_p\cdot p\cdot\frac{dE}{dx} - \mu_p\cdot E\cdot\frac{dp}{dx} + D_p\cdot\frac{d^2p}{dx^2} + G_p - R_p$$

Some silicon parameters

- Bandgap (indirect) : E_g(300 K)~1,12 eV
- Effective density of states (conduction band) : 3,22 · 10^{19} cm^{-3}
- Effective density of states (valence band) : 1,83 · 10^{19} cm^{-3}
- Intrinsic carrier density n_i (300K) : 1,3 · 10^{10} cm^{-3}
- Intrinsic mobility of electrons (300K) ~ 1400 cm^2/Vs
- Intrinsic mobility of holes (300K) ~ 500 cm^2/Vs
- Indirect band gap of Si: 1.12 eV

ε_0 : 8.8 10^{-12} F/m

$\varepsilon_{r\ SiO2}$: 3.9

$\varepsilon_{r\ Si}$: 11.9

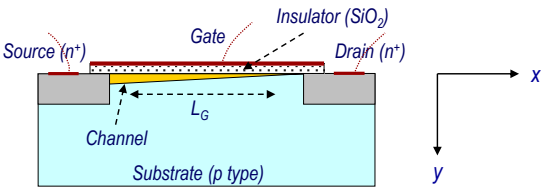
k (Boltzmann) = 1.38 10^{-23} J/K

I- Alternative modeling of the bulk MOSFET.

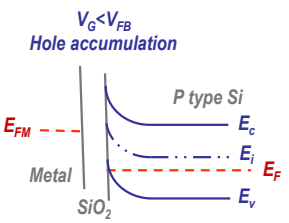
Recalling basics of MOSFETs
Traditional modelling

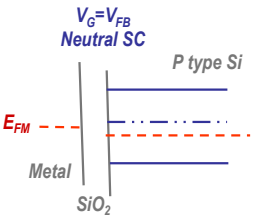
The MOSFET structure

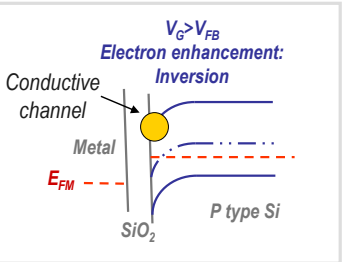
• Basic structure of a MOSFET.



• Depending on the gate potential with respect to the substrate, there are 3 modes of operation:







• The Flat Band voltage V_{FB} depends on the work function differences at the gate and substrate contacts, as well as on fixed charge densities in the SiO_2. *! V_{FB} is not the threshold voltage V_T !*

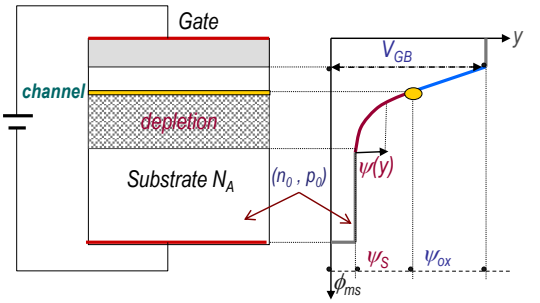
The MOSFET model

- Potentials drop inside a MOS structure.

ψ_S is the surface potential = the potential drop inside the semiconductor.

- If $V_G < V_{FB}$, then $\psi_S < 0$ (accumulation)

-If $V_G > V_{FB}$, then $\psi_S > 0$ (depletion inversion.)



- From Boltzmann statistics:

$$\begin{cases} n(y) = n_0 \cdot \exp\left(\dfrac{\psi(y)}{U_T}\right) \\ p(y) = p_0 \cdot \exp\left(-\dfrac{\psi(y)}{U_T}\right) \end{cases}$$

$p_0 = N_A$: Total acceptor ionization

n_0, p_0 : @ equilibrium, $n_o.p_o = n_i^2$

- Neutrality in the substrate imposes: $n_0 + N_A - p_0 = 0$ ⇒ $\rho(y) = q \cdot ([n(y) - n_0] - [p(y) - p_0])$

Poisson equation:

Gradual channel approximation:

2D electrostatics is decomposed in 2 x 1D problem

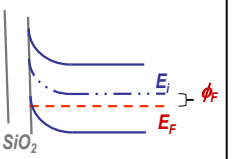
$$\frac{d^2\psi(y)}{dy^2} = -\frac{\rho(y)}{\varepsilon} = \frac{q}{\varepsilon} \cdot \left(p_0 \cdot \left(e^{-\frac{\psi(y)}{U_T}} - 1 \right) - n_0 \cdot \left(e^{\frac{\psi(y)}{U_T}} - 1 \right) \right)$$

The MOS capacitor

Assuming that $p_o \sim N_A$ [redacted]

According to the definition of the Fermi potential, we can write: $n_i / N_A = e^{\frac{-\phi_F}{U_T}}$

And noting that [redacted]



We can multiply the first relation by : $d\psi(y)/dy$

[redacted]

After some manipulations, and since the electric field and ψ vanish in the neutral region:

$$E(y) = -\frac{d\psi(y)}{dy} = \pm\sqrt{\frac{2 \cdot q \cdot N_A}{\varepsilon}} \cdot \sqrt{U_T \cdot e^{\frac{-\psi(y)}{U_T}} + \psi(y) - U_T + e^{\frac{-2\phi_F}{U_T}} \cdot \left(U_T \cdot e^{\frac{\psi(y)}{U_T}} + \psi(y) - U_T \right)}$$

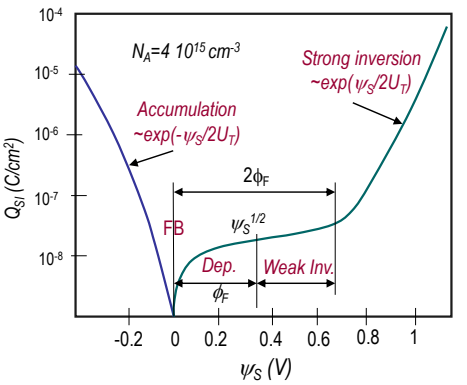
The gradual channel approximation is adopted to neglect the transverse electric field.

Charge density in MOS capacitor

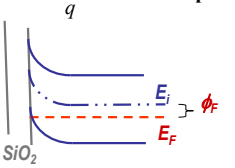
Gauss law: The flux of E across a closed surface = Q_{SC}/ε

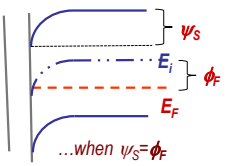
$$Q'_{Sc} = \mp\sqrt{2 \cdot q \cdot \varepsilon \cdot N_A} \cdot \sqrt{U_T \cdot e^{\frac{-\psi_S}{U_T}} + \psi_S - U_T + e^{\frac{-2\phi_F}{U_T}} \cdot \left(U_T \cdot e^{\frac{\psi_S}{U_T}} + \psi_S - U_T \right)}$$

Unlike an ideal planar capacitor, the charge stored in the semiconductor doesn't vary linearly with the electrostatic potential.

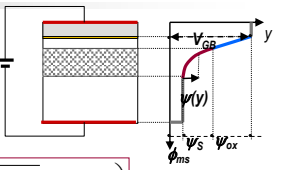


$$\phi_F = \frac{E_F - E_{Fi}}{q} = \textbf{Fermi pot.}$$





The inversion charge density



• 2 contributions under depletion-inversion mode:

We assume that the Depletion charge is : $Q_D' = -\sqrt{2 \cdot q \cdot \varepsilon \cdot N_A} \cdot \sqrt{\psi_S}$

And therefore the Inversion charge will be :

$$Q_I' = -\sqrt{2 \cdot q \cdot \varepsilon \cdot N_A} \cdot \left(\sqrt{\psi_S + U_T \cdot e^{\frac{\psi_S - 2 \cdot \phi_F}{U_T}}} - \sqrt{\psi_S} \right)$$

• The gate potential satisfies: $V_{GB} = \phi_{MS} + \psi_S + \psi_{OX} = \left(\phi_{MS} + \frac{Q_{Traps}}{C_{ox}'} \right) + \psi_S + \frac{Q_{Si}}{C_{ox}'}$

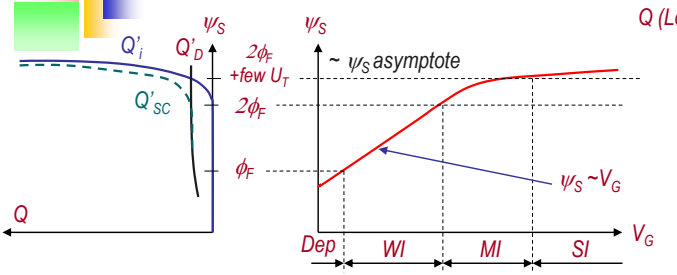
• In depletion-inversion mode, this simplifies into:

$$V_G = V_{FB} + \psi_S + \gamma \cdot \sqrt{\psi_S + U_T \cdot e^{\frac{\psi_S - 2 \cdot \phi_F}{U_T}}} \Rightarrow \psi_S(V_G) \Rightarrow Q_D(V_G), Q_i(V_G)$$

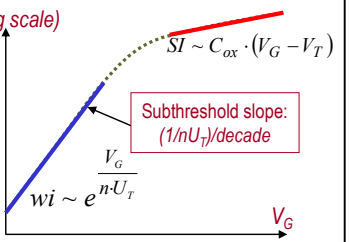
body factor $\left[\gamma = \frac{\sqrt{2q\varepsilon N_A}}{C_{OX}'} \right]$

flat band voltage, a process dependent parameter.

The threshold voltage, Weak and Strong inversion



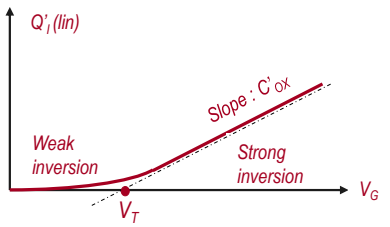
• In strong inversion, the surface potential reaches an asymptotic value ~$2\phi_F$ + few U_T



• Asymptotic expressions for Weak & Strong inversion.

• No such approximation for Moderate inversion !

• Derivation of the threshold voltage: $V_T = V_G^{(Q_I \approx 0 \text{ but SI})} = V_{FB} + 2\phi_F + \gamma \cdot \sqrt{2\phi_F}$



In a MOSFET, the threshold voltage will not only depend on the contact workfunctions, but also on the semiconductor doping.

The MOSFET: including the drain voltage

• Source and drain contacts will only affect the Fermi potential of electrons (P type sub.).

$$n(x,y)^{MOSCAP} = N_A \cdot \exp\left(\frac{\psi(x,y) - 2\cdot\phi_F}{U_T}\right) \Rightarrow n(x,y)^{MOSFET} = N_A \cdot \exp\left(\frac{\psi(x,y) - 2\cdot\phi_F - V(x)}{U_T}\right)$$

where V @source = V_S and V @drain = V_D

• In inversion, the depletion and inversion charges are then given by:

$$Q_B' = -\sqrt{-2\cdot q\varepsilon\cdot N_A}\cdot\sqrt{\psi_S}$$

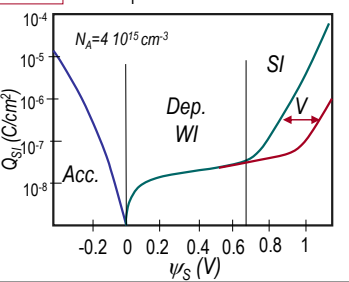
$$Q_I' = -\sqrt{2\cdot q\cdot\varepsilon\cdot N_A}\cdot\left(\sqrt{U_T\cdot e^{\frac{\psi_S - 2\phi_F - V}{U_T}} + \psi_S} - \sqrt{\psi_S}\right)$$

⇒ Implicit dependence of Q_B on V through Q_I

⇒ Explicit dependence of Q_I on V

• Changing the potential of the (N) source will mainly affect the concentration of electrons in the channel in strong inversion (for P type substrate).

• An increase in V will decrease the inversion charge density: at the origin for the saturation effect in MOSFETs (increase in V_D).



The channel current

- Accounting for drift and diffusion currents in a 1D model, we have:

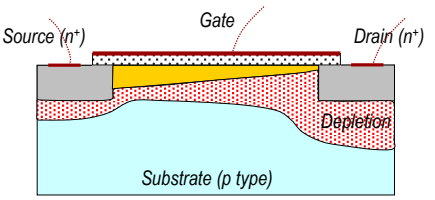
$$I = W \cdot \left(-Q_I'(\psi_S) \cdot \mu_n \cdot \frac{d\psi_S}{dx} + D_n \cdot \frac{dQ_I'(\psi_S)}{dx} \right) \quad \text{or} \quad I = -Q_I \cdot W \cdot \mu \cdot \frac{dV}{dx}$$

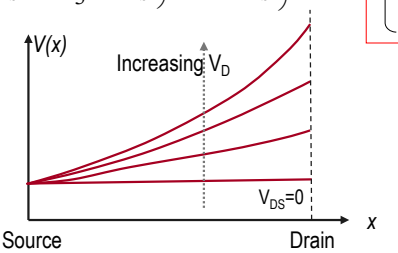
- Given $Q_I(\psi_S)$, after integration from source to drain and noting that I is constant with x:

$$I = \frac{W}{L} \cdot \left(-C_{OX}' \cdot \mu_n \cdot \left((V_G - V_{FB}) \cdot \psi_S \Big|_S^D - \frac{1}{2}\psi_S^2 \Big|_S^D - \gamma \cdot \frac{2}{3}\psi_S^{3/2} \Big|_S^D \right) + D_n \cdot Q_I' \Big|_S^D \right)$$

- The current only depends on source and drain surface potentials (or charges) which are respectively a function of (V_G,V_S) and (V_G, V_D) (no longer true with 'history effects,i.e. ferroelectrics).
- Having calculated the current, we can also integrate from the source to any x:

$$I = \frac{W}{x} \cdot \left(-C_{OX}' \cdot \mu_n \cdot \left((V_G - V_{FB}) \cdot \psi_S \Big|_S^X - \frac{1}{2}\psi_S^2 \Big|_S^X - \gamma \cdot \frac{2}{3}\psi_S^{3/2} \Big|_S^X \right) + D_n \cdot Q_I' \Big|_S^X \right) \Rightarrow \begin{cases} \psi_S(x) \\ Q_I(x) \\ V(x) \end{cases}$$

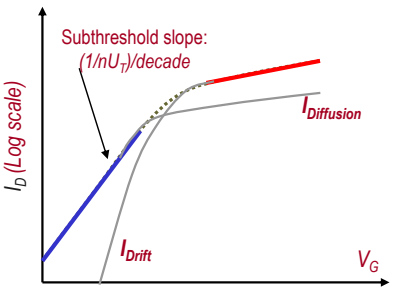




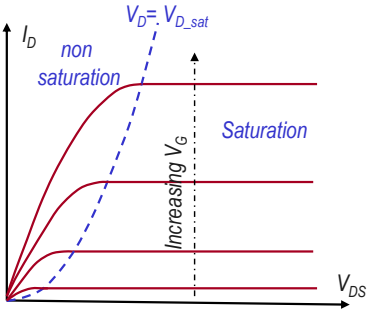
The channel current

- Typical I-V MOSFET characteristics (assuming V_S=0)

I_D versus V_G in Log scale



I_D versus V_D in Lin scale



- In weak inversion (& sat), the current $\sim e^{\frac{V_G}{n \cdot U_T}}$
- In strong inversion, saturation $\sim (V_G - V_T)^2$
- In strong inversion, conduction $\sim V_D \cdot \left(2 \cdot \frac{V_G - V_T}{n} + V_D\right)$

A new approach to model MOSFETs

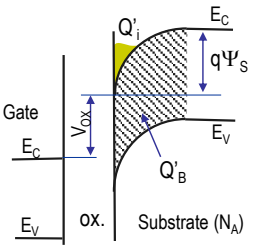
Charge Linearization Concept.

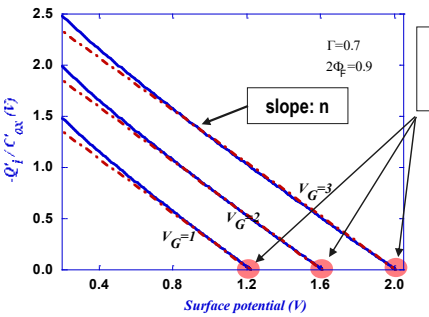
Potential drop and charge linearization: (the substrate is the reference)

$$V_G = V_{FB} + \psi_S - \frac{Q'_B}{C'_{OX}} - \frac{Q'_I}{C'_{OX}} \quad (V_{ox})$$

$$\frac{-Q'_B}{C'_{ox}} \approx \frac{\sqrt{2 \cdot q \cdot \varepsilon \cdot N_A}}{C'_{ox}} \sqrt{\psi_S}$$

$$V_G = V_{FB} + \psi_S + \Gamma \cdot \sqrt{\Psi_S} - \frac{Q'_I}{C'_{OX}}$$





pinch-off surface potential : $\Psi_P(V_G)=\Psi_S(q_i=0)$

⇒ The pinch-off surface potential depends on V_G

Slope factor: $n \approx 1 + \frac{\Gamma}{2 \cdot \sqrt{\Psi_P}}$

$\Phi_F = U_T \, \mathrm{Ln}(N_A/n_i)$ ~0.4V for 10^{17} cm^{-3}

$$\Psi_P = V_G - V_{FB} - \Gamma^2 \cdot \left(\sqrt{\frac{V_G - V_{FB}}{\Gamma^2} + \frac{1}{4}} - \frac{1}{2} \right)$$

The pinch-off surface potential and slope factor define a linear Q-V_G characteristic

Charge Linearization Concept.

- The dependence of Q_I with Ψ_S being almost linear for **fixed V_G**, we have:

$$\frac{Q_i'}{C_{ox}'} = n \cdot (\Psi_S - \Psi_P)$$

Where n is the slope factor :

$$n \approx 1 + \frac{\Gamma}{\sqrt{2 \cdot \Phi_F + \Psi_P}}$$

$\Phi_F = U_T Ln(N_A/n_i)$

$\sim (0.4V \text{ for } 10^{17}\ cm^{-3})$

*Note that this approximation relies 'only' on the **electrostatics** of the gate-dielectric system.*

- The slope factor can also be obtained from the equivalent gate capacitance:

$$\frac{Q_i'}{C_{ox}'} = n \cdot (\Psi_S - \Psi_P)$$

$$\frac{dV_G}{dQ_G} = \frac{d\psi_S}{dQ_G} + \frac{dV_{ox}}{dQ_G} = \left(\frac{-dQ_I}{d\psi_S} - \frac{dQ_B}{d\psi_S} \right)^{-1} + \frac{1}{C_{ox}} = \frac{1}{C_D - n \cdot C_{ox}} + \frac{1}{C_{ox}}$$

- **Maintaining V_G constant**, we obtain:

$$n = 1 + \frac{C_D}{C_{ox}}$$

C_D: depletion capacitance

Charge Linearization Concept.

- Then, we still have to see how this will impact the relation existing between Q_I and Ψ_S, which is now governed by the semiconductor.

$$Q_i' = -\Gamma \cdot C_{ox}' \cdot \sqrt{U_T} \cdot \left[\sqrt{\frac{\Psi_S}{U_T} + \exp\left(\frac{\Psi_S - 2 \cdot \Phi_F - V_{ch}}{U_T} \right)} - \sqrt{\frac{\Psi_S}{U_T}} \right]$$

$$\frac{\Psi_S}{U_T} = \frac{Q_i'}{n \cdot C_{ox}' \cdot U_T} + \frac{\Psi_P}{U_T} \quad \Rightarrow \quad \Downarrow$$

$$Ln\left[\frac{-Q_i'}{\Gamma \cdot C_{ox}' \cdot \sqrt{U_T}} \cdot \left(\frac{-Q_i'}{\Gamma \cdot C_{ox}' \cdot \sqrt{U_T}} + 2 \cdot \sqrt{\frac{Q_i'}{n \cdot C_{ox}' \cdot U_T} + \frac{\Psi_P(V_G)}{U_T}} \right) \right] - \frac{Q_i'}{n \cdot C_{ox}' \cdot U_T} = \frac{\Psi_P(V_G) - 2\Phi_F}{U_T} - \frac{V_{ch}}{U_T}$$

This represents a new relation between the mobile charge density and potentials.

The Pinch-Off Voltage

We can show that in strong inversion, the source and drain potentials are linked to the surface potential through:

$$V_{S,D} \overset{SI}{\approx} \psi_s^{S,D} - 2\cdot\phi_F$$

- Therefore, in strong inversion, the charge density at source and drain can also be written in terms of the source and drain potentials.

$$\frac{Q'_{S,D}}{n\cdot C'_{ox}} = \left[\psi_s^{S,D} - 2\phi_F\right] - \left[\Psi_P - 2\phi_F\right] \overset{SI,Lin}{\sim} V_{S,D} - \left[\Psi_P - 2\phi_F\right]$$

- Based on this relation, we define a 'pinch-off' voltage V_P as: $V_P = \Psi_P - 2\phi_F$

- In strong inversion, the relation between charges and potentials can then be written as:

$$-Q'_{i@D,S} \overset{SI}{\approx} n\cdot C_{ox}\cdot\left(V_p - V_{S,D}\right)$$

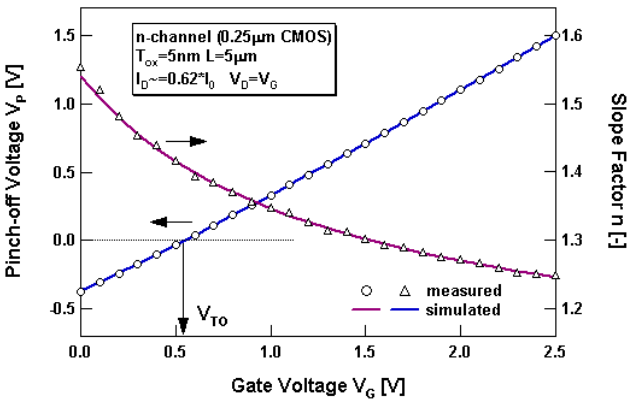
As for ψ_P, V_P will only depend on the gate voltage.

V_P is the gate potential 'seen' from the channel.

The Pinch-Off Voltage

A first order Taylor expansion of the V_P-V_G relation gives a useful approximation of the pinch-off voltage for 'hand calculations'.

$$V_p \approx \frac{V_G - V_{T0}}{n}$$



The pinch-off voltage and the slope factor can be measured !

V_P and n can be measured !

$$n \approx 1 + \frac{\Gamma}{2 \cdot \sqrt{2 \cdot \Phi_F + V_P}}$$

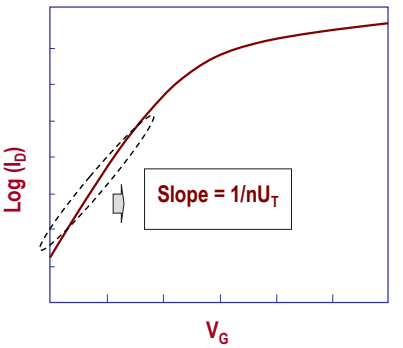
Note that n is also the slope of the Log (I_D) vs V_G characteristic in weak inversion (V_P~0) that equals *1/n* U_T . It varies between 1.1 (strong inversion) up to 1.6 (weak inversion).

What about weak-inversion ?

As for strong inversion, we can show that in weak inversion, charge densities at source and drain are still a function of the difference between V_P and V_S or V_D:

$$Q_{S,D} = -2 \cdot n \cdot C_{ox} \cdot U_T \cdot e^{\frac{V_P - V_{S,D}}{U_T}}$$

- *'n'* is also the slope of the Log (I_D) vs V_G characteristic in weak inversion that equals $1/nU_T$
  - *n* varies between 1.1 (strong inversion) up to 1.6 (weak inversion).



- Without demonstration, the general relation between charges and potentials is :

$$Ln\left(\frac{-Q_{S,D}}{2 \cdot n \cdot C_{ox} \cdot U_T}\right) - 2 \cdot \frac{Q_{S,D}}{2 \cdot n \cdot C_{ox} \cdot U_T} = \frac{V_P - V_{S,D}}{U_T}$$

Some more steps…

$$Ln\left[\frac{-Q_i'}{\Gamma\cdot C_{ox}'\cdot\sqrt{U_T}}\cdot\left(\frac{-Q_i'}{\Gamma\cdot C_{ox}'\cdot\sqrt{U_T}}+2\cdot\sqrt{\frac{Q_i'}{n\cdot C_{ox}'\cdot U_T}+\frac{\Psi_P(V_G)}{U_T}}\right)\right]-\frac{Q_i'}{n\cdot C_{ox}'\cdot U_T}=\frac{\Psi_P(V_G)-2\Phi_F}{U_T}-\frac{V_{ch}}{U_T}$$

Concerning the Log function, what matters is to be accurate when Q<<1

$$Ln\left[-\frac{Q_i'}{n\cdot C_{ox}'\cdot U_T}\cdot\left(\frac{2n\sqrt{\Psi_P(V_G)}}{\Gamma}\right)\right]-\frac{Q_i'}{n\cdot C_{ox}'\cdot U_T}=\frac{\Psi_P(V_G)-2\Phi_F}{U_T}-\frac{V_{ch}}{U_T}$$

$$Ln\left[-\frac{Q_i'}{n\cdot C_{ox}'\cdot U_T}\right]-\frac{Q_i'}{n\cdot C_{ox}'\cdot U_T}=\frac{\Psi_P(V_G)-2\Phi_F}{U_T}-\frac{V_{ch}}{U_T}+Ln\left[\frac{2n\sqrt{\Psi_P(V_G)}}{\Gamma}\right]$$

$$Ln\left[-\frac{Q_i'}{n\cdot C_{ox}'\cdot U_T}\right]-\frac{Q_i'}{n\cdot C_{ox}'\cdot U_T}\cong\frac{\Psi_P(V_G)-2\Phi_F}{U_T}-\frac{V_{ch}}{U_T}$$

Drain Current & normalization.

Charge linearization can also be used for the current.

- Adopting the drift-diffusion transport model:

$$I_D = \mu W(-Q_i' \frac{d\psi_S}{dx} + U_T \frac{dQ_i'}{dx}) \quad \Longrightarrow \quad I_D = \mu W(-Q_i' \frac{dQ_i'}{dx} \cdot \frac{1}{n \cdot C_{ox}} + U_T \frac{dQ_i'}{dx})$$

- Since I_d is constant along the channel, integration from S to D gives:

$$I_D \cdot L = \mu W \int_S^D \left(-\frac{Q_i}{nC_{Ox}} + U_T \right) \cdot dQ_i \quad \Longrightarrow \quad I_D = 2n\mu C_{OX} U_T^2 \frac{W}{L} \left(\left[\frac{Q_i}{2nC_{OX}U_T} \right] - \left[\frac{Q_i}{2nC_{OX}U_T} \right]^2 \right)_S^D$$

(assuming a constant mobility μ)

- We define a specific current I_{SP} and specific charge density Q_{SP} that depend only on the technological parameters:

$$I_{SP} = 2n\mu C_{OX} U_T^2 \frac{W}{L}$$

$$Q_{SP} = 2nC_{OX}U_T$$

- The current can be written:

$$\frac{I_D}{I_{SP}} = \left[\left(\frac{Q_S}{Q_{SP}} \right)^2 - \left(\frac{Q_S}{Q_{SP}} \right) \right] - \left[\left(\frac{Q_D}{Q_{SP}} \right)^2 - \left(\frac{Q_D}{Q_{SP}} \right) \right]$$

Drain Current & normalization.

- **The MOSFET can be modelled with only 2 normalized relations:**
  - For the current

$$i = \left(q_S^2 + q_S\right) - \left(q_D^2 + q_D\right)$$

  - For the charges

$$Ln(q_i) + 2 \cdot q_i \approx v_p - v_{ch}$$

$$\left.\begin{array}{l} Ln(q_S) + 2 \cdot q_S \approx v_p - v_S \longrightarrow q_S \\ Ln(q_D) + 2 \cdot q_D \approx v_p - v_D \longrightarrow q_D \end{array}\right\} \longrightarrow i$$

Drain Current & transconductances.

- Since the inversion charge densities at source and drain are always given by a general form involving the difference between V_P and the source and drain potentials V_S and V_D:

$$Q_D = F(V_P - V_D) \quad \text{and} \quad Q_S = F(V_P - V_S)$$

The current takes a simple form:

$$I_D = H(V_P - V_S) - H(V_P - V_D)$$

- This has implications in small signal analysis:
  - A variation δV of V_P is equivalent to a **simultaneous** variation $-\delta V$ of V_D and V_S

$$\frac{\partial I_D}{\partial V_G} = \frac{\partial I_D}{\partial V_P} \cdot \frac{\partial V_P}{\partial V_G} = \left(\frac{\partial I_D}{\partial(-V_S)} - \frac{\partial I_D}{\partial(-V_D)} \right) \cdot \frac{1}{n} \quad \Longrightarrow \quad \boxed{g_m = \frac{g_{ms} - g_{md}}{n}}$$

- In saturation, $g_{md} \sim 0$ and we obtain: $g_m \overset{sat}{\approx} \frac{g_{ms}}{n}$

The Inversion Factor IF.

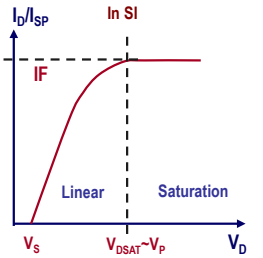
In saturation, the drain current doesn't change any more when increasing the drain voltage above V_{DSAT}, which means a negligible mobile charge at the drain:

$$\frac{I}{I_{SP}} \overset{sat}{\approx} \left[\left(\frac{Q_S}{Q_{SP}}\right)^2 - \left(\frac{Q_S}{Q_{SP}}\right)\right]$$

- Therefore, in strong inversion, V_{DSAT} is almost V_P.

$$-Q'_{i@D} \overset{SI,sat}{\approx} n \cdot C_{ox} \cdot \left(V_p - V_{Dsat}\right) \approx 0$$

  - *Note that V_D=V_G ensures saturation since V_D(=V_G) > V_P.*



- By definition, the inversion factor IF is the normalized current of the device operating in saturation:

$$IF \overset{SAT}{=} \frac{I}{I_{SP}}$$

$$-Q'_{i@S} \overset{SI,sat}{\approx} n \cdot C_{ox} \cdot \left(V_p - V_S\right)$$

$$\frac{I}{I_{SP}} \overset{SI,sat}{\approx} \left(\frac{Q_S}{Q_{SP}}\right)^2$$

in strong inv. ⇒

$$IF \overset{SI,SAT}{\approx} \left(\frac{V_P}{2U_T}\right)^2$$

IF 'holds for' V_G

The g_{ms}/I_D Invariant.

- An equivalent formulation of the current is given by:

$$I_D = \mu W(-Q_i')\frac{dV_{ch}}{dx} \quad \xrightarrow{\text{Integration along the channel}} \quad I_D = \int_{V_S}^{V_D} \beta \cdot \frac{-Q_i}{C_{ox}} \cdot dV$$

- Then, from the definition of the source transconductance, we obtain:

$$g_{ms} = -\frac{\partial I_D}{\partial V_S} = \beta\frac{-Q_S}{C_{ox}} = 2n\mu C_{ox}\frac{W}{L}U_T \cdot \left(\frac{-Q_S}{2nC_{ox}U_T}\right) = \frac{I_S}{U_T} \cdot \left(\frac{-Q_S}{2nC_{ox}U_T}\right)$$

- In addition, in saturation, Q_D ~0 and Q_S is related to the Inversion Factor:

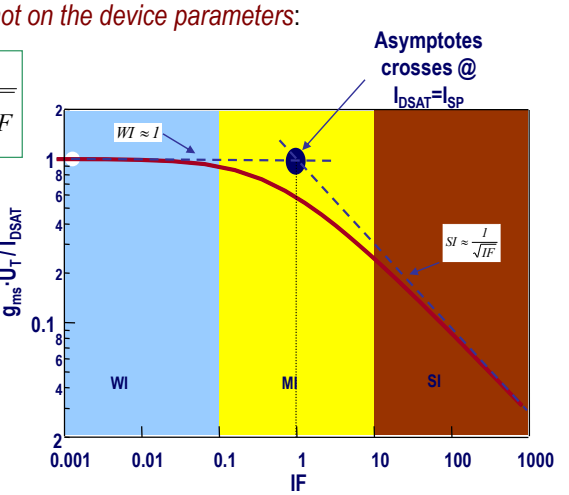
$$I_D^{SAT} = I_{SP} \cdot \left(\left[\frac{Q_S}{2nC_{OX}U_T}\right]^2 - \left[\frac{Q_S}{2nC_{OX}U_T}\right]\right) \quad \Longrightarrow \quad -\left[\frac{Q_S}{2nC_{OX}U_T}\right]^{SAT} = \frac{\sqrt{1+4\cdot IF}-1}{2}$$

The g_{ms}/I_D Invariant.

- In saturation, the source transconductance-to-current ration is only dependent on the inversion factor *IF, and not on the device parameters*:

$$\frac{g_{ms}}{I_{DSat}} \cdot U_T = n\frac{g_m}{I_{DSat}} \cdot U_T = \frac{1}{\frac{1}{2} + \sqrt{\frac{1}{4} + IF}}$$

The definition of I_{SP} (the current normalization factor) is consistent with the transition from weak to strong inversion, when the MOSFET operates in saturation.



Identifying velocity saturation on g_{ms}/I_D

- Assuming strong inversion and saturation, the drain current is given by:

[redacted] $\Rightarrow g_{m.} \overset{SI,SAT}{\approx} \sqrt{2\frac{\beta}{n_V} I_D} \Rightarrow \frac{g_{m.}}{I_D} \overset{SI,SAT}{\approx} \frac{1}{n_V U_T} \cdot \frac{1}{\sqrt{IF}}$

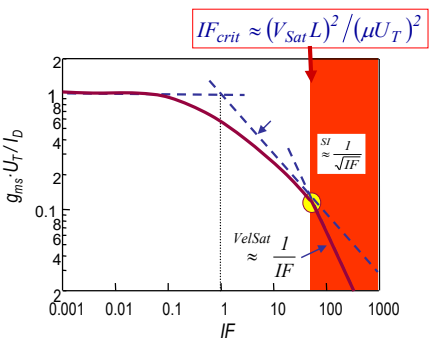
- At high V_D, velocity saturation will limit the current:

$$I_D \overset{Vel\_Sat}{\approx} (WC_{ox}(V_G - V_T)) \cdot V_{sat} \Rightarrow g_m \overset{Vel\_Sat}{\approx} WC_{ox} V_{sat} \Rightarrow \frac{g_{m.}}{I_D} \overset{SI,SAT}{\approx} \frac{V_{Sat} WC_{OX}}{I_{SP}} \cdot \frac{1}{IF}$$

- If V_{sat}=10⁵ ms⁻¹ & μ=0.05 m²V⁻¹s⁻¹

$IF_{crit}(L=1\mu m) \sim 64000$
$IF_{crit}(L=0.1\mu m) \sim 64$

Velocity saturation deteriorates g_{ms}/I_D characteristics.



What about multigate **U**ltra **T**hin **B**ody and **B**ulk **SOI** ?

Modern technologies are multigate (see next chapters)
Among them is the UTBB SOI MOSFET (ST Micro)

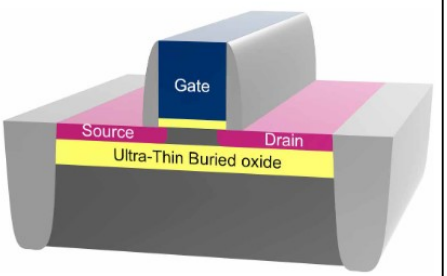
SOI = Silicon On Insulator
Used in more advanced technologies

The SiO_2 buried oxide can serve as a gate insulator, while the Si substrate is the gate electrode

Si active layer
SiO_2 buried oxide (100-10 nm)
Si substrate

When both the SiO_2 buried oxide and the Si active layer are in the nm range, the device can be controlled by 2 independent gates

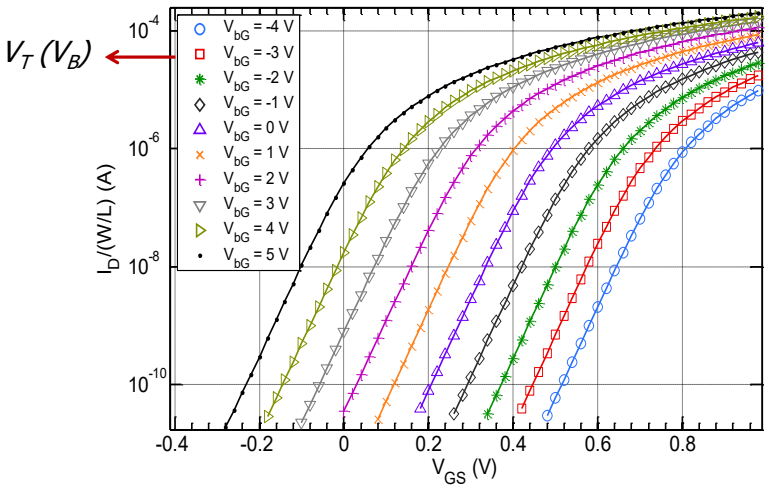
Silicon thickness : 7 nm
Front Gate insulator : High K , equivalent to 1.3 nm SiO_2
Back Gate insulator : 25 nm SiO_2
Independent front and back gates



https://www.st.com/content/st_com/en/about/innovation---technology/FD-SOI.html

The current depends on Back and Front Gate Voltages

The 'Front Channel' threshold voltage depends on the 'back gate' bias.



Multiple Threshold V_T is interesting for design optimization and flexibility:

- Standart-V_T circuits,

- Low V_T device for high-speed circuits

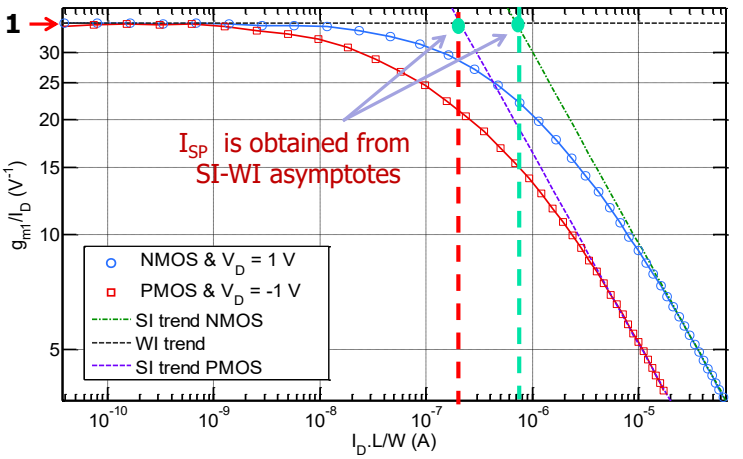
Transposing the Inversion Coefficient in UTBB SOI

❑ **Empirical determination of *n* & I_{SP} in UTBB SOI**

Determination of the specific curent and slope factor.

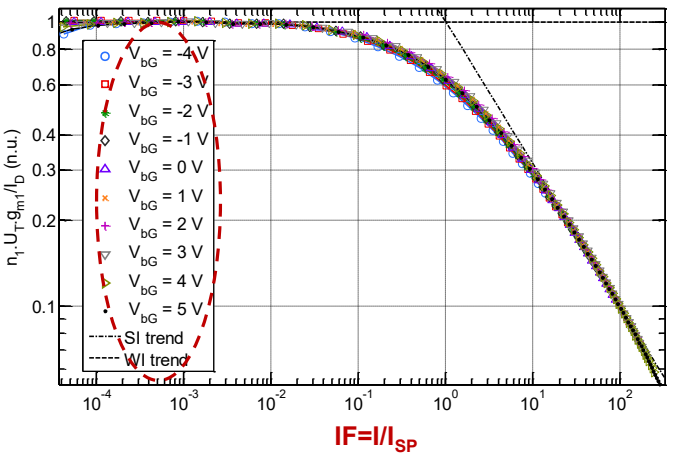
Front Gate Sweep, Back Gate grounded

n $g_m/I_D U_T = 1$ →



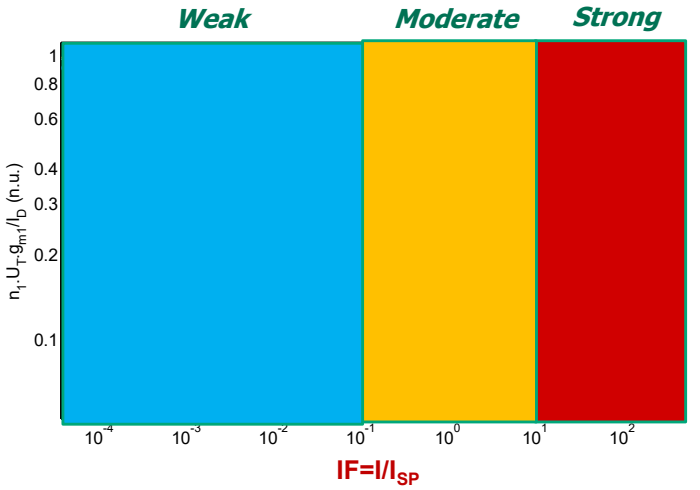
Transposing the Inversion Coefficient in UTBB SOI

After normalization, the invariance g_m/I versus I, based on experiments, is supported for different back gate biases.



Transposing the Inversion Coefficient in UTBB SOI

- ‘ Weak, Moderate and Strong Inversion in UTBB SOI

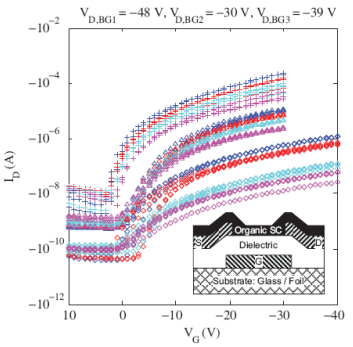


The ‘mode of operation’ is easily identified and becomes a concept independent of the gate voltages, only the channel current matters.

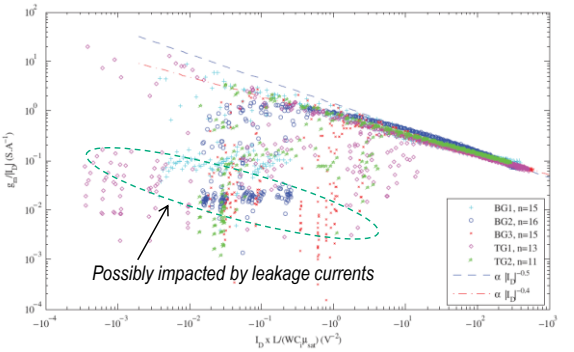
What about Organic FETs ?

• Organic electronic technology is less mature than silicon technology.
Devices are prone to have a quite large variability: makes circuit design more challenging.

Quite large variability of mobility and threshold voltages



g_m/I *versus* I/I_{sp}



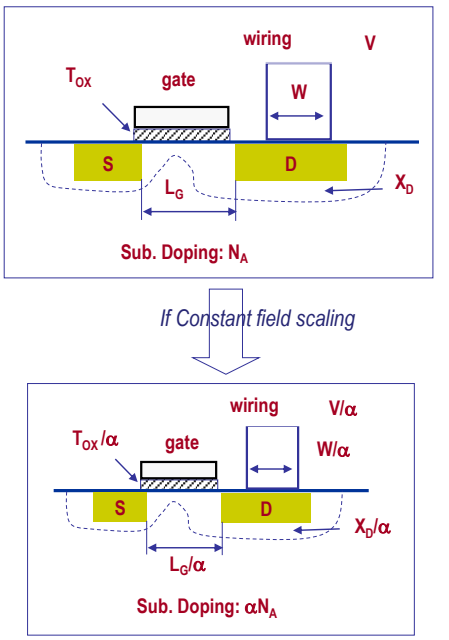
• Using the 'MOSFET' normalization for the current gives a more reliable representation in terms of g_m/I versus an 'estimated' normalization current.

Short channel effects in MOSFETs

Constant Field Scaling

- Scaling = change in physical parameters so that the scaled device has a similar behavior.
- A large FET is scaled down by a factor α (> 1) leading to a smaller FET that is expected to have similar behavior.
- Reducing voltages and dimensions by α and increasing the doping and charge densities by α leads to the same electric field distribution :
  constant field scaling

Time delay (CV/I) decreases in proportion to $1/\alpha$ and density in proportion to α^2



Jean-Michel SALLESE Slide 54

A large FET is scaled down by a factor □α□ to produce a smaller FET with similar behaviour. When all voltages and dimensions are reduced by □α□ , and when the doping and charge densities are increased by the same factor, the electric field inside the FET remains the same as in the original device. This is called constant field scaling. It results in circuit speed increasing in proportion to $1/\alpha$ and circuit density in α^2.

Constant Field Scaling

Scaling (constant field scaling – $\alpha > 1$)

Dimensions, L_G, W, T_{OX}	$1/\alpha$
Area	$1/\alpha^2$
Capacitances	$1/\alpha$
Capacitances per unit area	α
Devices per unit of chip area	α^2
Charges	$1/\alpha^2$
Doping concentrations	α
Voltages … and ideally also V_T	$1/\alpha$
Bias currents	$1/\alpha$
Transistor transit time	$1/\alpha^2$
Gate delay	$1/\alpha$
Power dissipation	$1/\alpha^2$

Limits to Scaling

- Limitations are mainly due to:
  - Nonscaling of the built-in potential (& junctions).
  - Non scaling of the subthreshold slope.
  - Non scaling of the threshold voltage.

  } Degradation of the OFF state current.

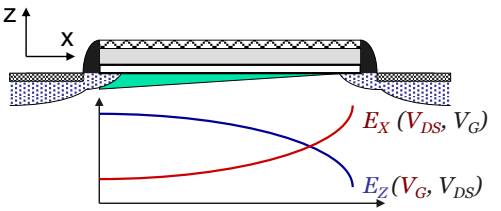
  - Quantum mechanical tunneling currents (gate to channel and source-dain).
  - Discrete nature of dopants in nm scaled devices: matching issues.

- Lowering of the nominal voltage down to 1 volt will also reduce the available dynamic range, pushing the devices to operate in weak-moderate inversion.

- Then, while scaling of CMOS technology improves digital applications, this evolution is rather detrimental for analog design since it introduces non-ideal characteristics.

Mobility Reduction

- Mobility is the key parameter that directly impacts the current density in a MOSFET. But moving to more advanced technologies may also degrade this quantity.
- Vertical and lateral electric fields alter the mobility in the channel.
  - With downscaling of the MOSFET, even though the supply voltage is decreased, electric fields are increased in strong inv./saturation.



Mobility is then voltage and position dependent

- Causes to mobility reduction due to the vertical field E_z:
  - Coulomb scattering μ_c: interaction with ionized impurities (at low field, high doping)
  - Phonon scattering μ_{ph} : interaction with lattice vibrations (at medium field)
  - Surface roughness μ_{rs} : roughness of the Si-SiO_2 interface (at high field)

Mobility reduction and velocity saturation will deeply affect I-V characteristics and introduce additional distortion.

Mobility will also affect charges, and so will impact the transcapacitances (~ second order effect).

There are mainly three different scattering mechanisms that affect the mobility.

All these scattering mechanisms are sensitive to the local electric field, and in particular to the vertical electric field when considering the non saturated region.

Note that the vertical field is decreased from source to drain, whereas the longitudinal field increases.

Mobility Reduction

- Mobility reduction due to the vertical field (*when* $E_x << E_z$):

  – The effect of vertical electric field is accounted through an effective field E_{eff} that accounts for the spatial distribution of inversion and depletion charges.

$$E_{eff} = \left| \frac{\eta \cdot Q_I' + Q_B'}{\varepsilon_{SC}} \right| \quad \begin{cases} \eta_n = 1/2 \\ \eta_p = 1/3 \end{cases}$$

- Low longitudinal field (E_X) mobility has 3 contributions:

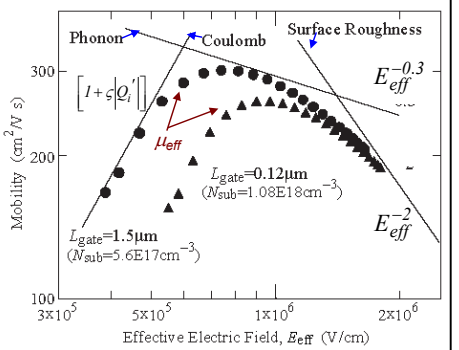
Coulomb scattering $\quad \mu_c \propto \left[1 + \varsigma \left| Q_i' \right| \right]$

Surface scattering $\quad \mu_{sr} \propto \left[E_{eff} \right]^{-2}$

Phonon scattering $\quad \mu_{ph} \propto \left[E_{eff} \right]^{-0.3}$

- These are combined through Mathiessen's rule:

$$\frac{1}{\mu_{eff}} = \frac{1}{\mu_c} + \frac{1}{\mu_{sr}} + \frac{1}{\mu_{ph}}$$

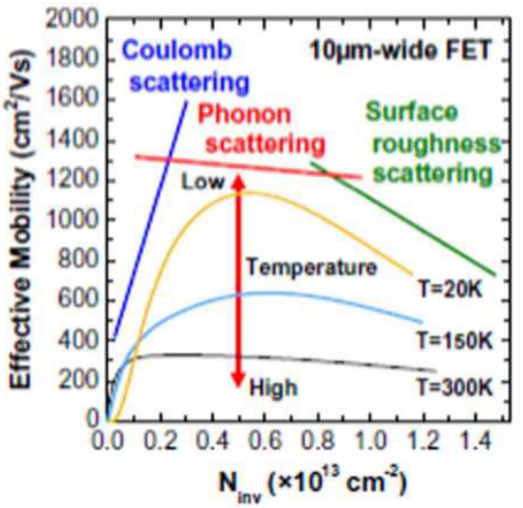


An effective vertical electric field governs the surface carrier mobility. It represents an average electric field that accounts for inversion and depletion charges. Except for the Coulomb mobility that increases with increasing Electric field, other contributions tend to decrease it.

Therefore, mobility depends both on gate and drain voltage.

At high electric fields the surface roughness scattering is the main cause to mobility degradation.

Since E_{eff} increases with scaling, we expect that the mobility will decrease: surface roughness scattering mechanims should be dominant in advanced CMOS devices.


Coulomb scattering
10μm-wide FET
Phonon scattering
Surface roughness scattering
Low
Temperature
High
T=20K
T=150K
T=300K
Effective Mobility (cm²/Vs)
N_inv (×10¹³ cm⁻²)
0
200
400
600
800
1000
1200
1400
1600
1800
2000
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Drain Induced Barrier Lowering (DIBL)

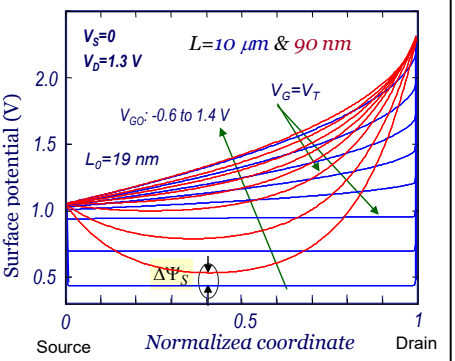
- In short channel devices, the longitudinal electric field (~V_{DS}) cannot be always neglected with respect to the normal component (~V_G), *particularly in weak inversion.*
- Electrostatic solution relies upon a quasi 2D description of the channel

DIBL lowers the surface potential in WI by $\Delta\psi_S$.
This can be seen as a decrease in V_{TO} *with* V_{DS} *&* V_G:

$$-\Delta V_{T0} \overset{wi}{\approx} -\Delta\Psi_S \approx \left(f(V_G)+V_D\right)\cdot\exp\left(\frac{-L_G}{2\cdot L_0}\right)$$

where L_o *is a characteristic length:*

$$L_0 = \eta(\approx 1)\cdot\sqrt{\frac{\varepsilon_{si}}{C_{ox}}h_{depl}} \sim \frac{1}{C_{ox}^{1/2}\cdot(N_{sub})^{1/4}}$$



- In weak inversion, short channel transistors are significantly affected by a high drain voltage. Reduction of DIBL requires thinner gate oxides and higher substrate doping.

Whereas in long channel devices the subthreshold current becomes independent of V_D as soon as it exceeds 4 U_T, for short channel I_D will continue to increase with V_D in weak inversion

This is due to the control of. the surface potential by the drain through a 2D effect since the lateral electric field can no longer be neglected.

These curves represent the surface potential along the channel calculated in weak inversion, both for a long and a short channel MOSFET.

In strong inversion, long and short channel surface potentials have almost the same spatial variation. This is no longer true in weak inversion.

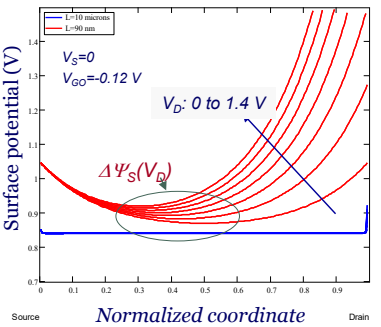
It is found that DIBL is governed by a Characteristic Length: when the channel length is much higher than L_o, the effect of DIBL can be neglected.

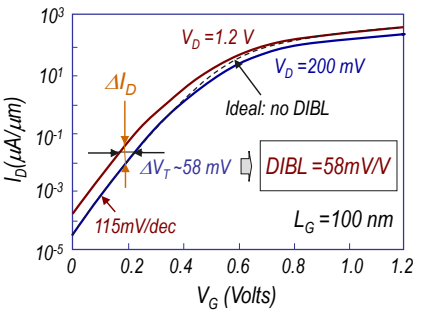
.

Drain Induced Barrier Lowering (DIBL)

- Lowering of the surface potential with V_D increases the current for short channel devices.
  - *Degradation of the output conductance g_{ds} , maximum voltage gain g_m/g_{ds} and I_{on}/I_{off}.*

$$\Delta\Psi_S(V_D) \sim V_D \Rightarrow \Delta I_D \sim \exp(\alpha V_D) \Rightarrow Ln(\Delta I_D) \sim \alpha V_D$$





(Degradation of Slope below the threshold)

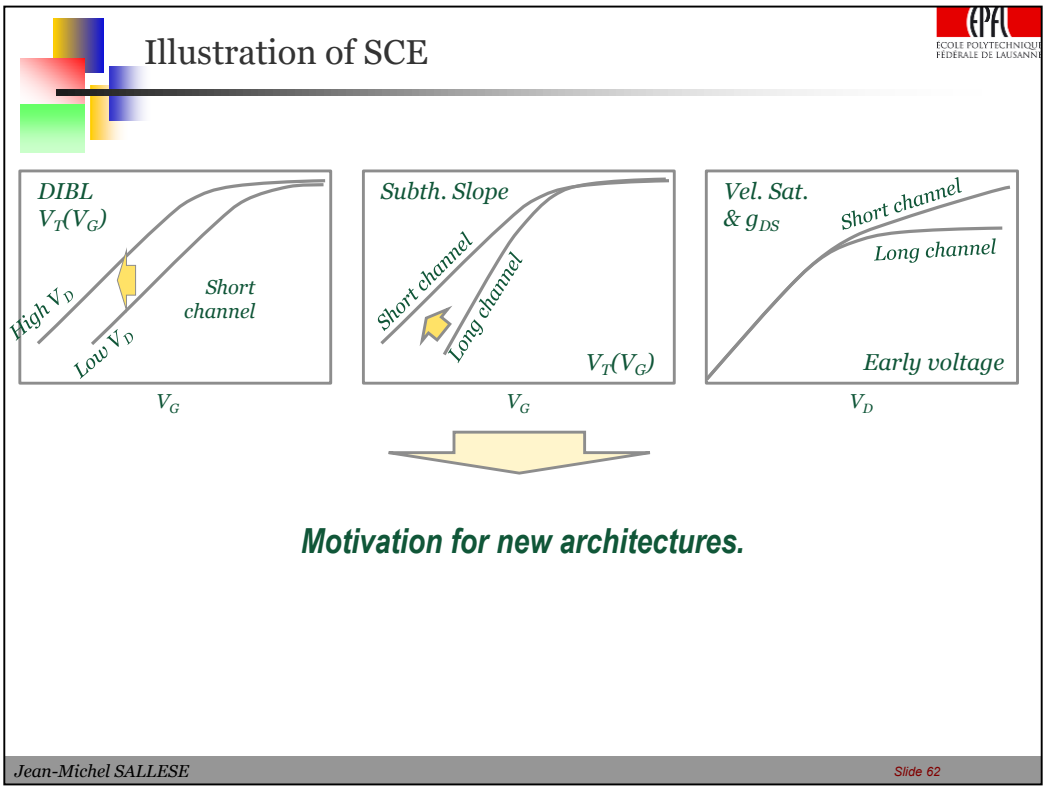
- DIBL is one of the most important short channel effect in weak inversion

In weak inversion, the surface potential of long channel MOFET's is almost independent of V_D when V_D >3UT.

In contrast, for short devices the surface potential will also depends on V_D : the maximum barrier seen from the source is lowered as V_D is increased, leading to the DIBL effect. This shift is almost linear with V_D.

Since in weak inversion the current depends exponentially on this shift, I_{OFF} will be degraded at high V_D.

To suppress source-drain leakage current, higher channel dopings are required. However, this is hardly compatible with high performances since it lowers mobility, enhances impact ionization and increases the threshold voltage (reducing the available overdrive voltage).



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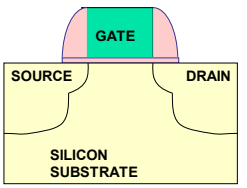
II- Modelling the Double Gate FETs.

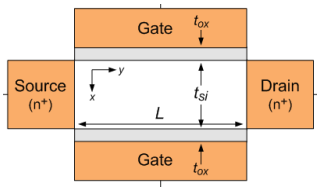
Why Double Gate MOSFETs ?

Bulk MOSFET



- Below 100 nm , bulk MOSFETs are difficult to control from the electrostatic point of view: DIBL, high WI slope
- Degradation of the Off state current, degradation of the dynamics, decrease in the current density.

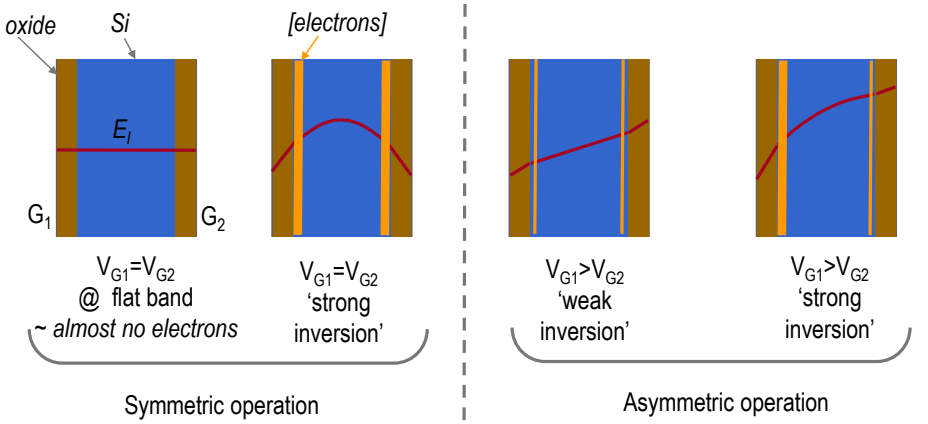
DG MOSFET



- **Advantages of double gate FETs:**
  - Better control from the gate
  - Almost ideal subthreshold slope ~ 60 mV/decade
  - Undoped silicon 'body, avoiding induced random dopant fluctuations, meaning 'less channel irregularities' and better device matching.

Different situations in a DG MOSFET

Depending on the potentials applied on the gates, there are different situations.



Not only non equal gate voltages will generate asymmetry. A difference in work functions and/or in gate oxide capacitances will also impact the symmetry.

The electrostatic solution in symmetric DG

Analytical solution for charges and current in the DG MOSFET.

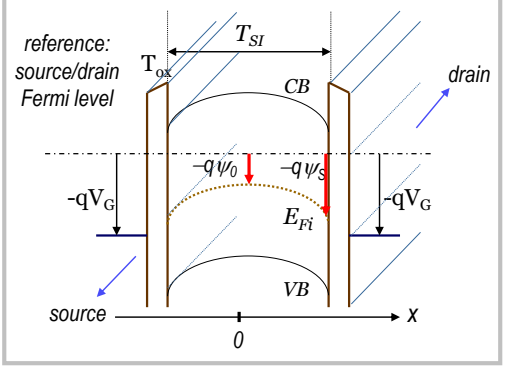
- Main assumptions are:

- Ideal 2 D structure .
- Quantum effects ignored.
- Boltzmann statistics in combination with Poisson equation (non degenerate)

The exact solution

The electrostatic solution in symmetric DG

Energy diagram



• Poisson equation (n type channel, holes neglected):

$$\frac{\partial^2\psi(x)}{\partial x^2} = -\frac{\left[Q_{mob} + Q_{fixed}\right]^{Undoped}}{\varepsilon_{si}} \approx \frac{q}{\varepsilon_{si}} \cdot n_i \cdot e^{\frac{\psi(x)}{U_T}}$$

• Integrating (symmetric solution):

$$\frac{\partial\psi(x)}{\partial x} = \sqrt{\frac{2kT \cdot n_i}{\varepsilon_{si}}} \cdot \sqrt{e^{\frac{\psi(x)}{U_T}} - e^{\frac{\psi(0)}{U_T}}} \quad \text{(for } x > 0\text{)}$$

ψ(0) is the 'potential' at the center.

• Likewise for the bulk MOSFET, the electric field is related to the potential drop *ψ(x)*.

• However, unlike the MOSFET where the electric field vanishes at infinity (and so for the potential), giving only *ψ(x)* will not determine *E(x)*…something is missing.

The electrostatic solution : 'detailed' derivation

Poisson equation (n type channel, holes can be neglected. E_F is constant, V_{DS}=0): $\frac{\partial^2\psi(x)}{\partial x^2} = -\frac{[Q_{mob}+Q_{fixed}]}{\varepsilon_{si}} \overset{Undoped}{\approx} \frac{q}{\varepsilon_{si}} \cdot n_i \cdot e^{\frac{\psi(x)}{U_T}}$

Can we use the same 'trick' as in MOSFET's ? Noting that:

$$\frac{\partial}{\partial x}\left(\frac{\partial\psi(x)}{\partial x}\right)^2 = \frac{\partial}{\partial x}E(x)^2 = 2\cdot\frac{\partial\psi(x)}{\partial x}\cdot\frac{\partial^2\psi(x)}{\partial x^2}$$

always >0

$$\frac{\partial}{\partial x}E(x)^2 = \frac{2\cdot q\cdot n_i}{\varepsilon_{si}}\cdot \mathbf{e}^{\frac{\psi(x)}{U}}\cdot\frac{\partial\psi(x)}{\partial x}$$

Then, we end up with: $\frac{\partial\psi(x)}{\partial x} = \pm\sqrt{\frac{2\cdot q\cdot n_i\cdot U_T}{\varepsilon_{si}}\cdot \mathbf{e}^{\frac{\psi(x)}{U}} + \bullet}$ ← *1st integration constant !*

Solving this differential equation requires special care…the type of solution will depend on C.

We need to make 2 changes of variables: $z(x) = \mathbf{exp}(\psi(x)/U_T)$ $\quad f(x) = \sqrt{z(x)+C}$

And we finally have to evaluate a primitive of the form: $\int \frac{1}{f^2 - C} df$

2nd integration constant !

• If C<0, then we can write : $C = -e^{\frac{\psi(0)}{U_T}}$

In that case, the general solution is $\psi(x) = \psi(0) - 2\cdot U_T \cdot \mathbf{ln}\left[\mathbf{cos}\left(e^{\frac{\psi(0)}{2\cdot U_T}}\cdot\sqrt{\frac{q\cdot n_i}{2\cdot\varepsilon_{si}\cdot U_T}}\cdot(x - x_0)\right)\right]$

$\psi(0)$ will represent the 'potential' at the center of the silicon film.

The electrostatic solution in symmetric DG

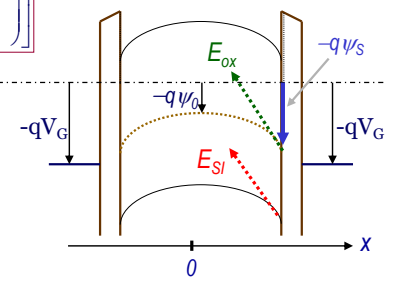
• Finally, the symmetry of the system with respect to x=o imposes x_0=0 :

$$\psi(x)-\psi(0)=-2\cdot U_T\cdot \ln\left[\cos\left(e^{\frac{\psi(0)}{2\cdot U_T}}\cdot\sqrt{\frac{q\cdot n_i}{2\cdot\varepsilon_{si}\cdot U_T}}\cdot x\right)\right]$$

always >0

• Setting x=$T_{SI}/2$ gives the potential at the channel interface (surface potential):

$$\psi_S=\psi(0)-2\cdot U_T\cdot \ln\left[\cos\left(e^{\frac{\psi(0)}{2\cdot U_T}}\cdot\sqrt{\frac{q\cdot n_i}{2\cdot\varepsilon_{si}\cdot U_T}}\cdot\frac{T_{Si}}{2}\right)\right]$$



So far, the solution is for the undoped silicon layer.

The gate also imposes a condition through capacitive coupling:

From the continuity of the displacement vector at the interface (no sheet charges), we must satisfy: $\varepsilon_{ox}\,E_{ox}=\varepsilon_{SI}\,E_{SI}$

The electrostatic solution in symmetric DG

• Given the surface potential, these electric fields are readily obtained:

$$-\varepsilon_{si}\cdot E_{SI}=\sqrt{2\cdot\varepsilon_{si}kT\cdot n_i}\cdot\sqrt{e^{\frac{\psi_s}{U_T}}-e^{\frac{\psi(0)}{U_T}}}$$

(note that electric fields 'are' <0 for x>0)

$$-\varepsilon_{ox}\cdot E_{ox}=\varepsilon_{ox}\cdot\frac{V_G-\Delta\phi-\psi_S}{T_{ox}}$$

$\Delta\phi$ is the work function difference between the gate electrode & the silicon. Its value is 0 for mid-gap electrodes, $-E_G/2q$ for N polygate, and $E_G/2q$ for P polygate

$$\varepsilon_{ox}\cdot\frac{V_G-\Delta\phi-\psi_S}{T_{ox}}=\sqrt{2\cdot\varepsilon_{si}kT\cdot n_i}\cdot\sqrt{e^{\frac{\psi_s}{U_T}}-e^{\frac{\psi(0)}{U_T}}}$$

and

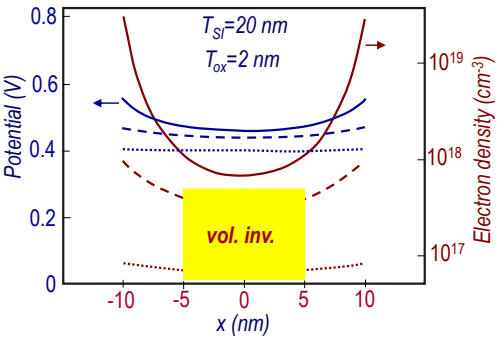
$$\psi_S=\psi(0)-2\cdot U_T\cdot\ln\left[\cos\left(e^{\frac{\psi(0)}{2U_T}}\cdot\sqrt{\frac{q\cdot n_i}{2\cdot\varepsilon_{si}\cdot U_T}}\cdot\frac{T_{Si}}{2}\right)\right]$$

We end up with two relations and 2 unknowns, $\psi(0)$ and ψ_S.
The system can then be solved for any gate voltage (but it must be consistent with electrons enhancement in the silicon film)

The electrostatic solution in symmetric DG

• Example of potential and electron density variations in the silicon for different gate potential.

V_G-$\Delta\phi$: 0.4 ; 0.57 ; 0.84



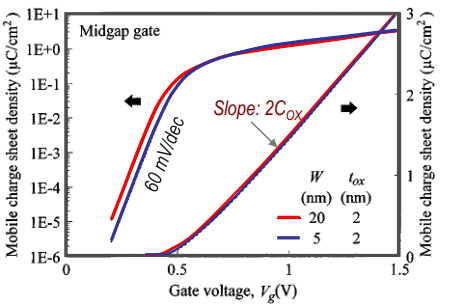
The total mobile charge density integrated across the silicon thickness, is obtained from the electric field at the 2 Si/SiO_2 interfaces:

$$Q_m = 2 \cdot C_{ox} \cdot (V_G - \Delta\phi - \psi_S)$$

$$n = n_i \cdot e^{\psi(x)/U_T}$$

In contrast to the bulk MOSFET where mobile charges are located at the Si/SiO_2 interface, carriers spread inside the silicon layer in DG MOSFET, leading to 'volume inversion'.

This is more pronounced at low concentrations ('weak inversion').



The electrostatic solution in symmetric DG

2 regions of operation:

- Exponential dependence at 'low' V_G
- Linear dependence at 'high' V_G

- Low V_G: neglecting the mobile charge, ψ_S follows the gate voltage: $\psi(0) \approx \psi_S \approx V_G - \Delta\phi$
Bands move as a whole and the mobile concentration is almost uniform: volume inversion

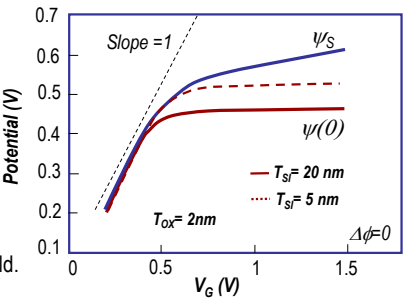
- High V_G: $\psi(0)$ reaches an asymptotic value since the cos function must be >0

$$e^{\frac{\psi(0)}{2U_T}} \cdot \sqrt{\frac{q \cdot n_i}{2 \cdot \varepsilon_{si} \cdot U_T}} \cdot \frac{T_{Si}}{2} < \frac{\pi}{2} \quad \Longrightarrow \quad \psi(0) \overset{\text{High } V_G}{\approx} U_T \cdot \ln\left(\frac{2 \cdot \varepsilon_{si} \cdot U_T \cdot \pi^2}{q \cdot n_i \cdot T_{Si}^{\,2}}\right)$$

The surface potential ψ_S still increases with V_G, but its value is almost independent of T_{SI}.

$$V_G - \Delta\phi \overset{SI}{\approx} \psi_S + \frac{\sqrt{2 \cdot \varepsilon_{si} kT \cdot n_i}}{C_{ox}} \cdot e^{\frac{\psi_S}{2U_T}}$$

$\psi(0)$ and Ψ_S are decoupled: no more volume inversion: the mobile charge @ Si/SiO_2 interface screens the gate electric field.



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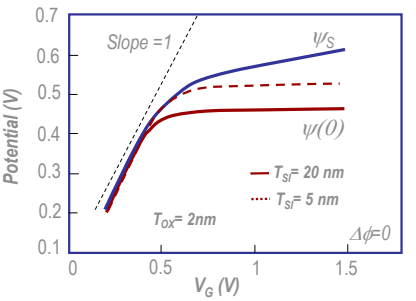
… for Bulk MOSFET we had …

$$V_G = V_{FB} + \psi_S + \gamma \cdot \sqrt{\psi_S + U_T \cdot e^{\frac{\psi_S - 2 \cdot \phi_F}{U_T}}}$$



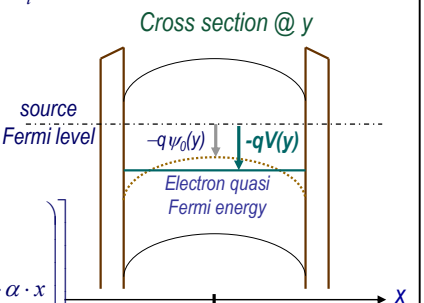
Current modelling of the DG MOSFET @ V_{DS}#0

- So far, we have derived the electrostatic solution of the DG MOSFET when V_{DS}=0. This approach will serve as a basis to develop analytical expressions of the current flowing through the device when V_{DS}#0.
- Neglecting the 'source to drain' electric field, Poisson equation can be rewritten:

$$\frac{\partial^2\psi(x,y)}{\partial x^2}+\frac{\partial^2\psi(x,y)}{\partial y^2}\approx\frac{\partial^2[\psi(x,y)-V(y)]}{\partial x^2}\overset{Undoped}{\approx}\frac{q}{\varepsilon_{si}}\cdot n_i\cdot e^{\frac{\psi(x,y)-V(y)}{U_T}}$$

V(y): Electron quasi Fermi potential @ y

The potential V(y) varies from source ($V=V_S$) to drain ($V=V_D$), but keeps a constant value in the x direction for a given y: Gradual channel approximation.



$$\psi(x,y)-V(y)=\psi_0(y)-V(y)-2\cdot U_T\cdot\ln\left[\cos\left(\frac{e^{\frac{\psi_0(y)-V(y)}{2\cdot U_T}}}{2\cdot U_T}\cdot\alpha\cdot x\right)\right]$$

At the centre, x=0

$$\alpha=\sqrt{\frac{2\cdot U_T\cdot q\cdot n_i}{\varepsilon_{si}}}$$

Current modelling of the DG MOSFET @ V_{DS}#0

The mobile charge density per unit area is obtained from the electric field @ SiO_2/Si **interfaces**:

$$Q_m(y) = 2 \cdot \varepsilon_{si} \cdot E_S(y) = -2 \cdot \varepsilon_{si} \cdot \frac{d\psi_S(x,y)}{dx} = -2 \cdot \varepsilon_{si} \cdot \frac{d}{dx}\left[\psi_S(x,y) - V(y)\right]$$

for the x>0 side

Or equivalently in terms of ψ_0:

$$Q_m(y) = -2 \cdot 2 \cdot U_T \cdot \varepsilon_{si} \cdot \alpha \cdot \frac{e^{\frac{\psi_0(y)-V(y)}{2U_T}}}{2 \cdot U_T} \cdot \tan\left(\frac{e^{\frac{\psi_0(y)-V(y)}{2U_T}}}{2 \cdot U_T} \cdot \alpha \cdot \frac{T_{si}}{2}\right)$$

Now, let us define a variable $a(y)$ such that

$$a(y) = e^{\frac{\psi_0(y)-V(y)}{2U_T}} \cdot \frac{1}{2 \cdot U_T} \cdot \alpha \cdot \frac{T_{si}}{2}$$

The mobile charge density becomes a simple function of a(y):

$$Q_m(y) = -8 \cdot U_T \cdot \frac{\varepsilon_{si}}{T_{si}} \cdot a(y) \cdot \tan(a(y))$$

Current modelling of the DG MOSFET @ V_{DS}#0

The total current, including drift and diffusion components, is still given by the relation:

$$I = -W \cdot \mu \cdot Q_m(y) \cdot \frac{dV(y)}{dy}$$

Integration along the channel →

$$I \overset{\mu\ cst}{=} -\frac{W}{L} \cdot \mu \cdot \int_S^D Q_m(y) \cdot \frac{dV(y)}{dy} \cdot dy$$

Introducing the variable $a(y)$, the drain current can be rewritten as:

$$I \overset{\mu\ cst}{=} -\frac{W}{L} \cdot \mu \cdot \int_S^D Q_m(y) \cdot \frac{dV(y)}{da(y)} \cdot da(y)$$

$$a(y) = e^{\frac{\psi_0(y) - V(y)}{2 \cdot U_T}} \cdot \frac{1}{2 \cdot U_T} \cdot \alpha \cdot \frac{T_{si}}{2}$$

... but we still have to link V(y) with a(y), or equivalently with ψ_o(y)...

$$V_G - \Delta\phi - \psi_S(y) = E_S(y) \cdot \frac{\varepsilon_{si}}{\varepsilon_{ox}} \cdot T_{ox} = \frac{-Q_m(y)}{2} \cdot \frac{T_{ox}}{\varepsilon_{ox}}$$ *'electrostatics'*

$$\psi_S(y) - V(y) = 2 \cdot U_T \cdot \ln\left(\frac{2 \cdot U_T}{\alpha} \cdot \frac{2}{T_{si}} \cdot a(y)\right) - 2 \cdot U_T \cdot \ln(\cos[a(y)])$$ *'SC statistics'*

$$V_G - \Delta\phi - [\text{illegible}] = 2U_T \cdot \ln\left(\frac{2U_T}{\alpha} \cdot \frac{2}{T_{si}} \cdot [\text{illegible}]\right) - 2U_T \cdot \ln(\cos[\text{illegible}]) + 4U_T \cdot \frac{T_{ox}}{T_{si}} \cdot \frac{\varepsilon_{si}}{\varepsilon_{ox}} \cdot a(y) \cdot \tan[\text{illegible}]$$

Current modelling of the DG MOSFET @ V_{DS}#0

At this point, we can note that such a relation will implicitly provide the values of a(y) at the source and drain once V_G, V_D and V_S are known.
This will be required when calculating I

$$a(y) = e^{\frac{\psi_0(y) - V(y)}{2 \cdot U_T}} \cdot \frac{1}{2 \cdot U_T} \cdot \alpha \cdot \frac{T_{si}}{2}$$

Derivating V(y) versus a(y) gives:

$$\frac{-dV(y)}{da(y)} \cdot \frac{1}{2U_T} = \frac{1}{a(y)} + 2\frac{T_{ox}}{T_{si}} \cdot \frac{\varepsilon_{si}}{\varepsilon_{ox}} \cdot \frac{a(y)}{\cos^2[a(y)]} + \tan[a(y)] \cdot \left(1 + 2\frac{T_{ox}}{T_{si}} \cdot \frac{\varepsilon_{si}}{\varepsilon_{ox}}\right)$$

We can now calculate the current integral. The final result is:

$$I \cdot \frac{L}{W} \cdot \frac{1}{\mu} \overset{\mu \text{ cst}}{=} \left(16U_T^2 \cdot a(y) \cdot \tan(a(y)) \cdot \frac{\varepsilon_{si}}{T_{Si}}\right) \cdot \left(1 + a(y) \cdot \tan(a(y)) \cdot \frac{T_{ox}}{T_{Si}} \cdot \frac{\varepsilon_{si}}{\varepsilon_{ox}}\right) - 8\frac{\varepsilon_{si}}{T_{si}} \cdot U_T^2 \cdot a(y)^2 \Big|_S^D$$

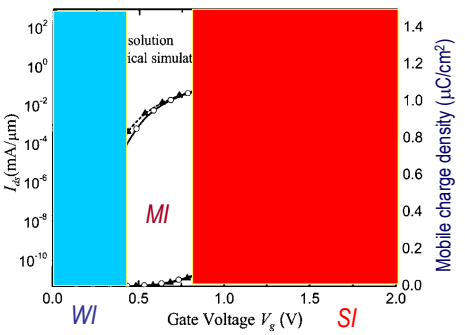
The integral of $Q_m(y) \cdot dV(y)/da(y)$ over a(y) only depends on the value of a(y) at the limits which can be evaluated from V_G, V_S and V_D.

The current will only depend on source and drain nodes, just like for the bulk MOSFET.

Current modelling of the DG MOSFET @ V_{DS}#0

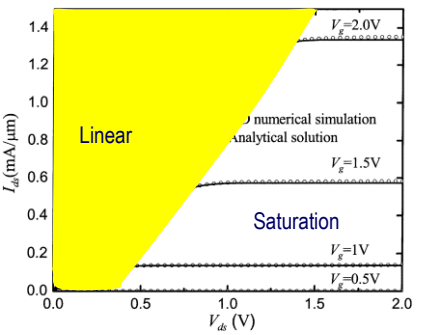
Comparison with 2D numerical simulations confirms the validity of the assumptions

I_D versus V_G



Likewise for the bulk MOSFET, we can identify different regimes that correspong to weak, moderate and strong inversion operation.

I_D versus V_D



Likewise for the bulk MOSFET, we can identify linear and saturation zones.

Current modelling of the DG MOSFET @ V_{DS}#0

Asymptotic relations.

- 'Strong Inversion': Remember that when the gate voltage exceeds a certain value (which would correspond to a threshold voltage not yet defined), we found the following condition:

$$e^{\frac{\psi(0)}{2U_T}} \cdot \sqrt{\frac{q \cdot n_i}{2 \cdot \varepsilon_{si} \cdot U_T}} \cdot \frac{T_{Si}}{2} = a(y) \approx \frac{\pi}{2}$$

$$I \cdot \frac{L}{W} \cdot \frac{1}{\mu} \overset{\mu csti}{=} \left(16U_T^2 \cdot a(y) \cdot tan(a(y)) \cdot \frac{\varepsilon_{si}}{T_{Si}} \right) \cdot \left(1 + a(y) \cdot tan(a(y)) \cdot \frac{T_{ox}}{T_{Si}} \cdot \frac{\varepsilon_{si}}{\varepsilon_{ox}} \right) - 8 \frac{\varepsilon_{si}}{T_{si}} \cdot U_T^2 \cdot a(y)^2 \Bigg|_S^D$$

$$V_G - \Delta\phi - V(y) = 2U_T \cdot ln\left(\frac{2U_T}{\alpha} \cdot \frac{2}{T_{si}} \cdot a(y) \right) - 2U_T \cdot ln(cos[a(y)]) + 4U_T \cdot \frac{T_{ox}}{T_{si}} \cdot \frac{\varepsilon_{si}}{\varepsilon_{ox}} \cdot a(y) \cdot tan[a(y)]$$

In that case, the dominant term in the charge-potential and current-a(S,D) relations comes from the *tangent* function.

$$V_G - \Delta\phi - 2U_T \cdot \mathbf{ln}\left(\frac{4U_T}{\alpha} \cdot \frac{1}{T_{si}} \cdot \right) - V(y) \overset{\text{'SI'}}{\approx}$$

$-V_T$

$$I \cdot \frac{L}{W} \cdot \frac{1}{\mu} \overset{\text{'SI'}}{\approx} \left(\right)^2 \cdot \frac{\varepsilon_{ox}}{T_{ox}} \Bigg|_S^D$$

(This assumes that the drain is also in SI, i.e. no pinch-off)

$$I \overset{\text{'SI, Lin'}}{\approx} \mu \cdot \frac{W}{L} C_{ox} \left[(V_G - V_T)^2 - (V_G - V_T - V_{DS})^2 \right]$$

Cancels in saturation

Almost similar to bulk MOSFET ! T_{SI} does not enter in the expression

Current modelling of the DG MOSFET @ V_{DS}#0

Asymptotic relations.

- 'Weak Inversion': In that case, the parameter a(y) evaluated at source and drain contacts is very small. Then, the Log term will dominate in the charge-potential relation:

$$a(y) \overset{WI}{\approx} \frac{\alpha T_{SI}}{4U_T} \exp\left(\frac{V_G - V_T - V(y)}{2U_T}\right)$$

$$I \cdot \frac{L}{W} \cdot \frac{1}{\mu} \overset{\mu\,cst}{=} \left(16U_T^2 \cdot a(y) \cdot tan(a(y)) \cdot \frac{\varepsilon_{si}}{T_{Si}}\right) \cdot \left(1 + a(y) \cdot tan(a(y)) \cdot \frac{T_{ox}}{T_{Si}} \cdot \frac{\varepsilon_{si}}{\varepsilon_{ox}}\right) - 8\frac{\varepsilon_{si}}{T_{si}} \cdot U_T^2 \cdot a(y)^2 \Bigg|_S^D$$

- For the current, tangent functions can be neglected:

$$I \cdot \frac{L}{W} \cdot \frac{1}{\mu} \overset{WI}{\approx} -8\frac{\varepsilon_{si}}{T_{si}} \cdot U_T^2 \cdot a(y)^2 \Big|_S^D$$

$$V_G - \Delta\phi - V(y) = 2U_T \cdot ln\left(\frac{2U_T}{\alpha} \cdot \frac{2}{T_{si}} \cdot a(y)\right) - 2U_T \cdot ln(cos[a(y)]) + 4U_T \cdot \frac{T_{ox}}{T_{si}} \cdot \frac{\varepsilon_{si}}{\varepsilon_{ox}} \cdot a(y) \cdot tan[a(y)]$$

- Finally, the drain current in weak inversion can be expressed in terms of applied potentials:

$$\alpha = \sqrt{\frac{2 \cdot U_T \cdot q \cdot n_i}{\varepsilon_{si}}}$$

$$I \overset{'WI'}{\approx} \mu \cdot \frac{W}{L} \cdot (U_T \cdot q \cdot n_i \cdot T_{SI}) \cdot \exp\left(\frac{V_G - V_T}{U_T}\right) \cdot \left(1 - \exp\left(\frac{-V_{DS}}{U_T}\right)\right)$$

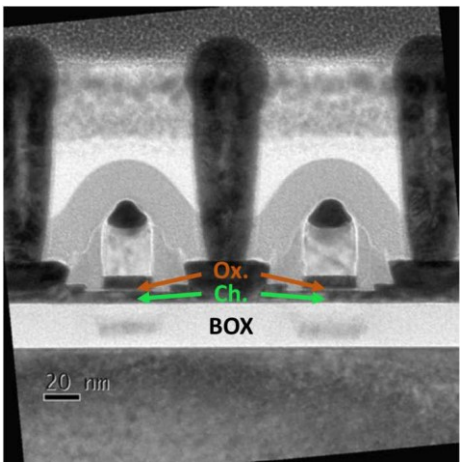
Ideal slope : 60 mV/Decade

Still very close to bulk MOSFET ! However, in weak inversion even though the current is independent of C_{OX} , it clearly depends on T_{SI} : volume inversion

The charge-based model of the double-gate FET

TEM image of an FD-SOI transistor

ultra-thin body and BOX (UTBB) FD-SOI CMOS,



28nm node with an high-k dielectric with an 1.1 nm equivalent oxide thickness for the front-gate oxide and with an ultra-thin (7 nm) conduction film positioned on top of a 25 nm BOX insulation layer.

Charge based model of DG FETs: Charges

- The model presented so far requires tedious iterative solutions and fails at identifying key parameters. We propose to develop an approximate solution that will lead to very compact formulations of all electrical quantities.
- Remember that, after integrating Poisson equation, we obtained the electric field across the silicon film:

$$E(x,y) = \pm\sqrt{\frac{2\cdot e\cdot U_T\cdot n_i}{\varepsilon_{si}}}\cdot\sqrt{e^{\frac{\psi(x,y)-V(y)}{U_T}} + C_1(y)}$$

Where $C_1(y)$ is the integration constant that was assigned to $-e^{\frac{\psi_0(y)}{U_T}}$ in the former approach…
but that we aim at keeping undetermined at this stage of the derivation.

Noting that the charge density per unit surface for each gate $Q_G(y)$ is obtained from $E\left(\frac{T_{Si}}{2}\right)$

$$\left.\begin{aligned} E\left(\frac{T_{Si}}{2}\right) &= \frac{Q_G(y)}{\varepsilon_{si}} \\ \psi_S(y) &= \psi_S(T_{Si}/2, y) \end{aligned}\right\} \Rightarrow \psi_S(y) - V(y) = U_T\cdot\ln\left(\frac{{Q_G}^2(y)}{2\cdot\varepsilon_{si}\cdot e\cdot U_T\cdot n_i} - C_1(y)\right)$$

In addition, the charge on the gate is also linked to the surface potential from electrostatics:

$$V_G - V_{FB} - \psi_S(y) = \frac{Q_G(y)}{C_{OX}}$$

Charge based model of DG FETs : Charges

- An implicit relation of the charge density on each gate is obtained once C_1 is known :

$$V_G - V_{FB} - V(y) = \frac{Q_G(y)}{C_{ox}} + U_T \cdot \ln\left(\frac{Q_G{}^2(y)}{2 \cdot \varepsilon_{si} \cdot e \cdot U_T \cdot n_i} - C_1(y)\right)$$

The integration constant $C_1(y)$ will then have a negligible impact on the charge density above the threshold, where the logarithmic term is negligible. Since $C_1(y)$ is representative of the electrostatic coupling between the gates, we expect such a coupling to be relatively small above the threshold.

Conversely, the situation is totally different below the threshold since the logarithmic term dominates, implying that a rather strong coupling should now occur between the two gates: 'volume inversion'.

Charge based model of DG FETs : Charges

- Finding C_1 requires a 'trick'....

We start from (if x>0): $E(x) = -\sqrt{\frac{2 \cdot e \cdot U_T \cdot n_i}{\varepsilon_{si}}} \cdot \sqrt{e^{\frac{\psi(x)-V(y)}{U_T}} + C_1(y)}$

Rearranging the solution

Imposed by symmetry

$$\sqrt{e^{\frac{\psi(x)-V(y)}{U_T}} + C_1(y)} = \sqrt{-C_1(y)} \cdot \tan\left[(x + Cst) \cdot \sqrt{\frac{2 \cdot e \cdot U_T \cdot n_i}{\varepsilon_{si}}} \cdot \frac{\sqrt{-C_1(y)}}{2 \cdot U_T}\right]$$

We recognize ' E(x) '

$$\psi(x) - \psi(0) = -2 \cdot U_T \cdot \ln\left[\cos\left(e^{\frac{\psi(0)}{2U_T}} \cdot \sqrt{\frac{q \cdot n_i}{2 \cdot \varepsilon_{si} \cdot U_T}} \cdot x\right)\right]$$

setting x to $T_{SI}/2$ and noting that $E(T_{SI}/2) = E_S = \varepsilon_{sI} Q_G$

$$Q_G(y) = \sqrt{2 \cdot \varepsilon_{si} \cdot e \cdot U_T \cdot n_i} \cdot \sqrt{-C_1(y)} \cdot \tan\left(\sqrt{\frac{2 \cdot e \cdot U_T \cdot n_i}{\varepsilon_{si}}} \cdot \frac{1}{2 \cdot U_T} \cdot \left(\frac{T_{Si}}{2}\right) \cdot \sqrt{-C_1(y)}\right)$$

We obtain a relation where the integration constant $C_1(y)$ and Q_G are correlated....
....but $C_1(y)$ is important only in weak inversion ...

Charge based model of DG FETs : Charges

- Assuming Q_G small enough, a first order approximation is obtained :

$$Q_G(y) \overset{wi}{\approx} \sqrt{2\cdot\varepsilon_{si}\cdot e\cdot U_T\cdot n_i}\cdot\sqrt{-C_1(y)}\cdot\left(\sqrt{\frac{2\cdot e\cdot U_T\cdot n_i}{\varepsilon_{si}}}\cdot\frac{1}{2\cdot U_T}\cdot\left(\frac{T_{Si}}{2}\right)\cdot\sqrt{-C_1(y)}\right)$$

$$\Longrightarrow \boxed{C_1(y)\approx -\frac{Q_G(y)}{e\cdot n_i\cdot\frac{T_{Si}}{2}}}$$

The integration constant takes a simple form

- The charge density on each gate can be computed from the difference between the gate and channel potentials:

$$\boxed{[V_G - V(y)] - V_{FB} = \frac{Q_G(y)}{C_{ox}} + U_T\cdot\ln\left(\frac{{Q_G}^2(y)}{2\cdot\varepsilon_{si}\cdot e\cdot U_T\cdot n_i} + 2\cdot\frac{Q_G(y)}{e\cdot n_i\cdot T_{Si}}\right)}$$

Charge based model of DG FETs : Normalization

- The relation between Charges and Potentials can be rearranged by identifying common factors, i.e. normalization factors:
  - Charges can be normalized: $q = Q / 4 C_{OX} U_T$
  - *Potentials can be normalized:* $v = V / U_T$

$$v_G - v(y) - v_{FB} + \ln\left(\frac{q_{\text{int}}}{2}\right) = 4 \cdot q_g(y) + \ln\left(q_g(y)\right) + \ln\left(1 + q_g(y) \cdot \frac{C_{ox}}{C_{Si}}\right)$$

q_{int} is intrinsic normalized charge density per unit surface: $q_{\text{int}} = \left(e \cdot n_i \cdot T_{Si}\right) / \left(4 \cdot C_{ox} \cdot U_T\right)$

C_{Si} is the silicon capacitance: $C_{Si} = \varepsilon_{si} / T_{Si}$

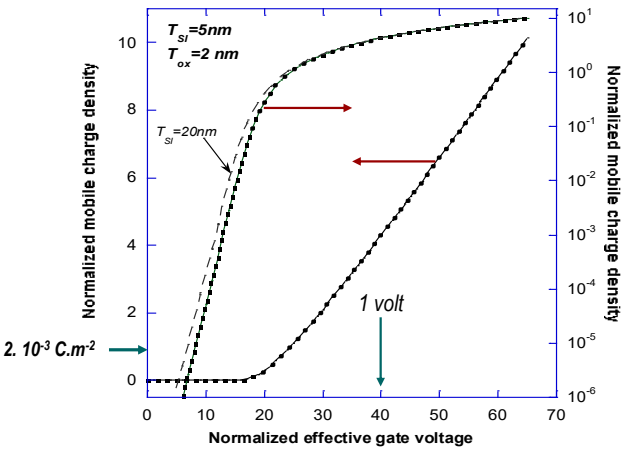
- The Threshold voltage in symmetric DG MOSFET's is obtained from the strong inversion asymptote that cancels the mobile charge:

$$v_T = v_{FB} - \ln\left(\frac{q_{\text{int}}}{2}\right) \quad \xrightarrow{\text{without normalization:}} \quad V_T = V_{FB} - U_T \cdot \ln\left(\frac{q_{\text{int}}}{2}\right)$$

The Threshold Voltage also depends on the silicon and oxide thicknesses (in addition to the material work functions).

Charge based model of DG FETs : Normalization

- Comparison between exact analytical solution and approximate solution



(here, the normalized charge factor = 2. 10^{-3} C m^{-2})

The mobile charge density is twice the charge on each gate (with opposite sign):

$$Q_m(y) = -2 \cdot Q_G(y)$$

In weak inversion, volume inversion is responsible for the increase of mobile charge with T_{SI}

Charge based model of DG FETs : Current

- Assuming drift-diffusion transport, the current in the channel is given by:

$$I = -\mu \cdot W \cdot Q_m(y) \cdot \frac{dV(y)}{dy} \quad \xrightarrow{I \text{ independent of } \boldsymbol{y} \text{ (quasi-static)}} \quad I = -\mu \cdot \frac{W}{L} \cdot \int Q_m(y) \cdot \frac{dV(y)}{dy} \cdot dy$$

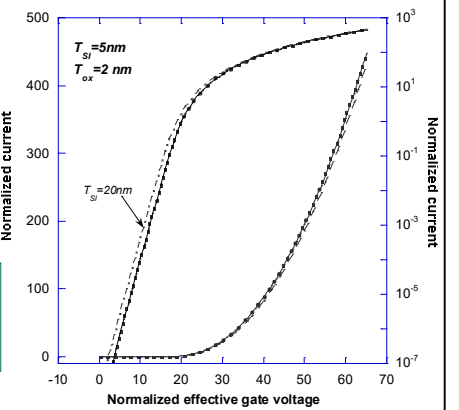
- Adopting normalized quantities, we obtain:

$$\frac{I}{4 \cdot \mu \cdot C_{ox} \cdot U_T^2 \cdot \frac{W}{L}} = i = -\int q_m(y) \cdot dv(y)$$

Specific current

Expressing dV as a function of Q_m and dQ_m, we get:

$$i = -q_m^2 + 2 \cdot q_m + 2 \cdot \frac{C_{Si}}{C_{ox}} \cdot \ln\left(1 - q_m \cdot \frac{C_{ox}}{2 \cdot C_{Si}}\right)\Bigg|_{q_m(S)}^{q_m(D)}$$



Charge based model of DG FETs : g_m/I invariant

- The transconductance to current ratio, evaluated in saturation, is an important parameter for analog design.
- The transconductance is given by : $g_m = \frac{dI}{dV_G} \overset{sat}{=} \frac{-dI}{dV_S} = -\mu \cdot \frac{W}{L} \cdot Q_{ms}$
- The transconductance to current ratio is then:

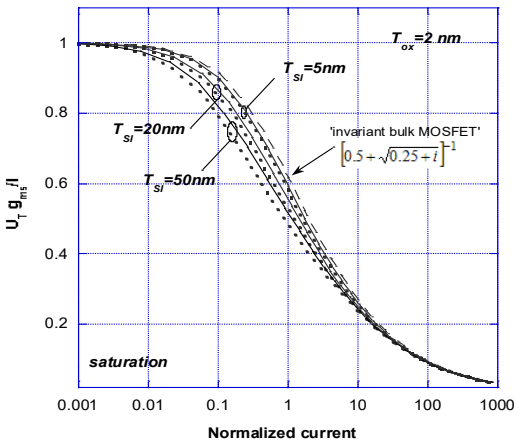
$$G(i) = \frac{g_m}{I} \cdot U_T = \frac{-q_{ms}}{q_{ms}^2 - 2 \cdot q_{ms} - 2 \cdot \frac{C_{si}}{C_{ox}} \cdot \ln\left(1 - 2 \cdot q_{ms} \frac{C_{ox}}{2 \cdot C_{si}}\right)}$$

Charge based model of DG FETs : g_m/I invariant

- An approximation of the current in weak inversion leads to:

'Bulk' 'DG'

$$\frac{1}{G(i)} \approx \left(\frac{1}{2}+\sqrt{\frac{1}{4}+i}\right) + \left(1+\frac{2\cdot C_{si}}{C_{ox}\cdot\left(\frac{1}{2}+\sqrt{\frac{1}{4}+i}\right)}\cdot \ln\left(1+\frac{C_{ox}}{2\cdot C_{si}}\cdot\left(\sqrt{\frac{1}{4}+i}-\frac{1}{2}\right)\right)\right)$$



This characteristic is almost independent of the devices parameters (T_{OX}, T_{SI}…):

Same design strategies apply for Symmetric DG and Bulk MOSFETs.

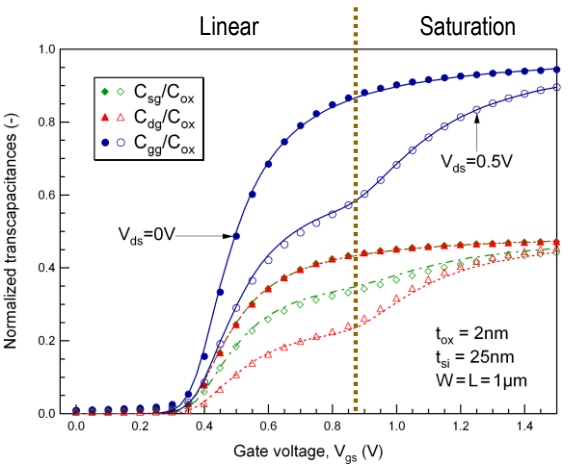
Charge based model of DG FETs : Capacitances

Transcapacitances as a function of V_{GS} at different V_{DS}. *Symbols: 2D; lines: analytical model.*

The relations linking the charge densities to the potentials have strong similarities with those of bulk MOSFET.

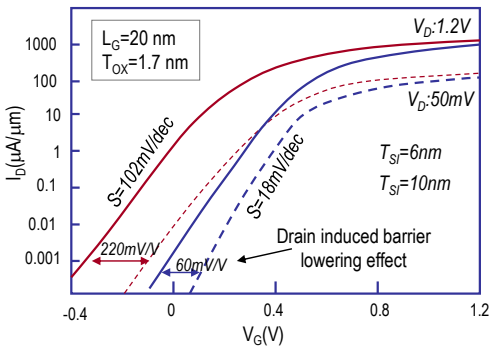
$$C_{ij} = \partial Q_i / \partial V_j$$

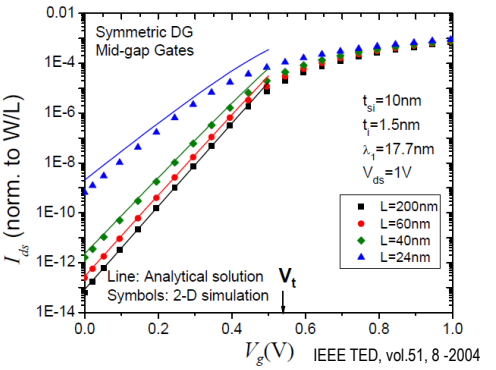
Partitioning of the mobile charge density between the source and the drain defines source and drain equivalent charges that are used for AC modelling



Short Channel Effects in DG FETs

- In a 20 nm channel length
  - The subthreshold slope is 102 mV/dec for T_{si}=10 nm and 81 mV/dec for T_{si}=6 nm
  - Drain Induced Barrier is 220mV/V for T_{si}=10 nm and 60 mV/V for T_{si}=6 nm.





IEEE TED, vol.51, 8 -2004

These departures from ideal 1D charateristics are due to short channel effects.

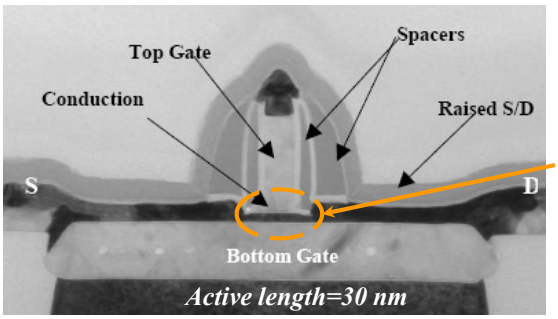
Even though there are two gates, some rules have to be defined between channel length and layers thicknesses (T_{ox} and T_{si}).

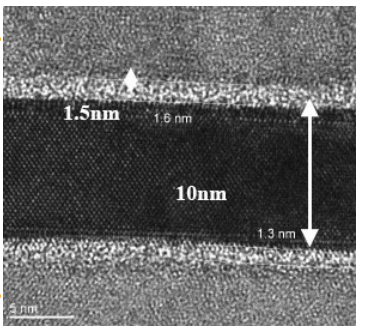
Quantum confinement in double-gate FET

Quantization effects in DG FETs

Emerging FET devices have dimensions comparable to the electron (hole) wavelength

Quantum corrections





- Depending on T_{SI}, 2D discrete levels cannot be ignored.
  These will modify the apparent band gap of the semiconductor and will affect the charge-potential dependence…but not only:
  The electric field in the silicon film will also impact the solution of the Schrodinger equation through the electric potential.
- The solution requires solving quantum mechanics AND electrostatics self-consistently.

Quantization effects in DG FETs

Typical values for DG MOSFET: T_{ox}=1 nm – T_{si}=10 nm – L_G < 20 nm

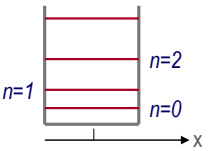
Is the behavior still "classical" ? Do carrier move effectively in a potential that varies 'slowly' ? Can we still use 3D DOS ? Is drift-diffusion still representative of the transport ? Are Boltzmann statistics accurate enough ?...

Adopting a 'rule of thumb' approach, we can 'intuitively' state that:

- For very thin silicon films (<5nm), 2D states are so confined that electrostatic correction can be ignored, but not the confined levels.
  This requires using a 2D density of states ?
- For relatively thick films (>20 nm), 2D levels can be ignored, reverting to the more classical description presented so far.
  We can still fairly use a 3D density of states ?
- For intermediate thicknesses, 2D confined states should be coupled to electrostatic.
  Should we use '2D+discrete levels' or '3D + continuum of states' ?

Quantization effects in DG FETs

- For an infinite confined quantum well, the solution of the Schrodinger equation leads to the following energy states and eigenfunctions:



$$\varphi_n(x) = \sqrt{\frac{2}{T_{Si}}} \cdot \cos\left(2 \cdot (n+1) \cdot \frac{x}{T_{Si}} - 1\right) \cdot \frac{\pi}{2} \qquad E_n = (n+1)^2 \frac{\pi^2 \cdot \hbar^2}{2 \cdot m \cdot {T_{Si}}^2}$$

- But what happens in a semiconductor when 'internal' potentials are changed ?

Can we still use the band structure of the semiconductor or do we have to recalculate it including the potential perturbation ?

We introduce 2 major approximations.

- We assume that the effective-mass approximation is valid, meaning that we can neglect the periodic potential and use instead the effective masses
- We assume that the envelope wave function (the wavefunction varies slowly with respect to the periodic atomic potential) vanishes at the Si/SiO_2 interface

These solutions presupposes that the wavefunction varies slowly with respect to the periodic atomic potential. This is known as the 'enveloppe function' approximation.

Are the two equations above sufficient to calculate the mobile charge density ?

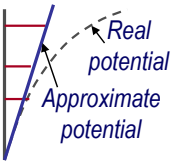
Is the density of state to be used 3D or 2D ?

Ex: Quantization effects in bulk MOSFETs

- In case of a bulk MOSFET and in weak-moderate inversion, the potential at the channel interface can be approximated by a triangular well. Solutions to the Schrodinger equation are then given in terms of Airy functions, leading to the following energy levels:

$$E_n \approx \left(\frac{\hbar^2}{2\cdot m}\right)^{\frac{1}{3}} \cdot \left(\frac{3}{2}\cdot\pi\cdot\frac{Q_{Si}}{\varepsilon_{Si}}\cdot\left(n+\frac{3}{4}\right)\right)^{\frac{2}{3}}$$

Stern PRB , vol.163, n.3 1967



- Under higher gate voltage, large carriers density will also affect the 'ideal' triangular well through the Poisson equation. The potential which appear in the Hamiltonian is obtained from the solution of the Poisson equation:

$$\begin{cases} \dfrac{-\hbar^2}{2m^*}\cdot\dfrac{d^2\psi_n(x)}{dx^2} - qV(x)\cdot\psi_n(x) \overset{\text{level 'n'}}{=} E_n\cdot\psi_n(x) \\ \dfrac{d^2V(x)}{dx^2} = \dfrac{q}{\varepsilon_{Si}}\cdot(N_A+n(x)) = \dfrac{q}{\varepsilon_{Si}}\cdot\left(N_A+\sum_n N_n\cdot|\psi_n(x)|^2\right) \overset{\substack{1^{ST}\text{ level occupied} \\ \approx \\ \text{Undoped}}}{\approx} \dfrac{-Q_m}{\varepsilon_{Si}}\cdot|\psi_0(x)|^2 \end{cases}$$

$$N_n = \mathbf{N_C}\cdot\int F(\varepsilon - E_n - E_F)\cdot d\varepsilon$$

No exact solutions exist in this case. The method consists in solving these coupled equations by choosing the 'best' trial wave function and minimize the related energy through a given parameter. This is known as the 'variational approach'.

N(x) is weighted by the

amplitude squared of the wave function

These solutions presupposes that the wavefunction varies slowly with respect to the periodic atomic potential. This is known as the 'enveloppe function' approximation.

Are the two equations above sufficient to calculate the mobile charge density ?

Is the density of state to be used 3D or 2D ?

Ex: Quantization effects in bulk MOSFETs

We choose $\psi_0(x)$ as a trial wave function for the fundamental confined energy level will minimize the energy of the confined state:

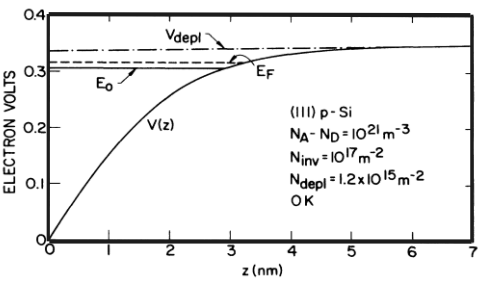
- In bulk MOSFETs, the ground state trial function for channel quantization is chosen such as :

$$\psi_0(x) = 2 \cdot a^{\frac{3}{2}} \cdot x \cdot \mathbf{e}^{-a \cdot x}$$

→ E_0 will depend on 'a' → Find 'a' that will minimize the energy E_0.

Fang, PRL , vol.16, 1966

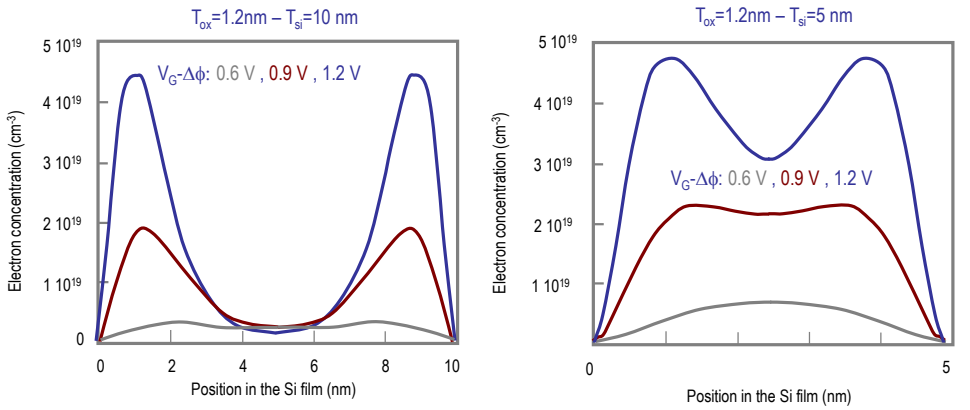
bulk MOSFET



Stern PRB , vol.163, n.3 1967

Quantization effects in DG FETs

Carrier concentration profile obtained by analytical modeling for various gate voltages for a silicon film thickness of 10 nm, 5 nm and 3 nm.



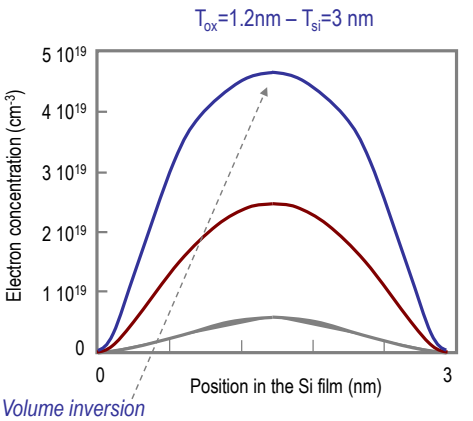
These curves are representative of the probability to find an electron, and thus are ~ $|\varphi(x)|^2$

In 'thick' silicon layers, the wave function is clearly affected by the carrier concentration through the electrostatic potential in the quantum well.

Quantization effects in DG FETs

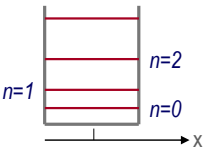
For the 3 nm case, the spatial distribution of the carrier concentration remains the same: The wave function is not affected by the charge density. This is expected for a very narrow quantum wells.

The inversion takes place at the center of the silicon film, even under high density ! …This was not expected with the classical model, even though it could predict volume inversion – *but in weak inversion only !*



Quantization effects in DG FETs

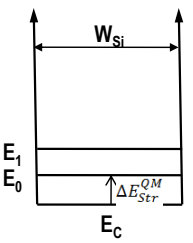
- For an infinite confined quantum well, the solution of the Schrodinger equation leads to the following energy states and eigenfunctions:



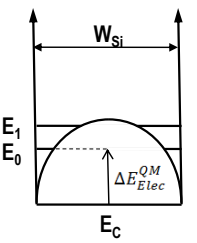
$$\varphi_n(x) = \sqrt{\frac{2}{T_{Si}}} \cdot \cos\left(2 \cdot (n+1) \cdot \frac{x}{T_{Si}} - 1\right) \cdot \frac{\pi}{2} \qquad E_n = (n+1)^2 \frac{\pi^2 \cdot \hbar^2}{2 \cdot m \cdot {T_{Si}}^2}$$

- But what happens in a semiconductor when 'internal' potentials are changed ?

$$\Delta E_0^{QM} = \Delta E_{Str}^{QM} + \Delta E_{Elec}^{QM}$$



Structural confinement



Electrical confinement

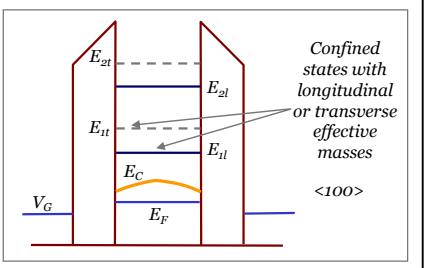
Quantization effects in DG FETs

- In order to take into account both the confinement due to the quantum well formed by the silicon film between the two gate dielectrics and the confinement induced by the electric field in strong inversion, we propose to rely on the variationnal approach:

- The trial wave function for the fundamental state must be symmetric with respect to the centre of the film

$$\varphi(u) \propto cosh(a \cdot u) \cdot cos\left(\frac{\pi}{2} \cdot u\right)$$

with $u=2x/T_{SI} - 1$, equal to 0 at the middle of the film



- It reduces to a *cosine function* in the weak inversion regime (a is expected to be small) since the potential energy is then close to that of a square-well.

- It reduces to a kind of $x \cdot exp(-ax)$ expression at each interface in the strong inversion regime, like for the bulk MOSFET.

In this approach, only one energy level is considered.

Quantization effects in DG FETs

$$\langle E_0 \rangle \overset{SI}{\approx} \frac{\hbar^2 \cdot \pi^2}{2 \cdot m^* \cdot T_{Si}^{\,2}} + \frac{3 \cdot 4^{2/3}}{8} \cdot \left(\frac{\hbar^2}{m^*} \right)^{1/3} \cdot \left(\frac{q \cdot Q_{inv}}{\varepsilon_{si}} \right)^{2/3}$$

Confinement by the potential well.

Confinement what would be obtained for a triangular well where the confinement is due to the transverse electric field

This energy increase in the fundamental state of the DG 'quantum well' can be seen as an increase in the threshold voltage, as if the semiconductor band gap was increased.

- Then, for a given surface potential ψ_s, the inversion charge calculated by including the quantum confinement would correspond to that of a surface potential $\psi_s - E_0/q$
- The apparent threshold voltage is then increased upon quantum confinement corrections.

But then, can we still use a 3D density of states ?
This will depend on the energy separation between the different confined levels:

- If E_i-E_{i+1} <U_T, then we can consider the system as 3D and we can safely use Boltzmann statistics (if non degenerated) with 3D DOS.

- If E_i-E_{i+1} >U_T, the system becomes 2D: we have to include exited states in the calculation of carrier density. In addition, we should also use Fermi Dirac statistics.

Quantization effects in DG FETs

$$\langle E_0 \rangle \overset{SI}{\approx} \frac{\hbar^2 \cdot \pi^2}{2 \cdot m^* \cdot T_{Si}^{\,2}} + \frac{3 \cdot 4^{2/3}}{8} \cdot \left(\frac{\hbar^2}{m^*} \right)^{1/3} \cdot \left(\frac{q \cdot Q_{inv}}{\varepsilon_{si}} \right)^{2/3}$$

Confinement by the potential well.

Confinement that would be obtained for a triangular well where the confinement is due to the transverse electric field

These quantum corrections can be included analytically:

- **New charge-potentials relationship:**

$$v_{gN}^{QM} - v_{to} - v_{ch} = 4 \cdot q_g + \ln q_g + \ln\left(1 + \alpha \cdot q_g\right) + \mathrm{A^{QM}} \cdot q_m^{2/3}$$

- **Normalized drain current expression:**

$$i = -{q_m}^2 + 2 \cdot q_m + \frac{2}{\alpha} \cdot \ln\left(1 - \frac{\alpha}{2} \cdot q_m\right) + \frac{2}{5} \cdot \mathrm{A^{QM}} \cdot q_m^{5/3} \Big|_{q_{mS}}^{q_{mD}}$$

No empirical parameter is needed

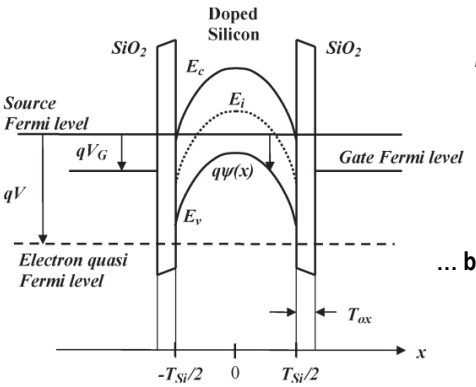
'only ' and additionnal term in the drain current expression

Including doping in DG FET

The doped DG MOSFET and the Equivalent Thickness concept

In practice, the doping can be used to tune the threshold voltage

Energy diagram of an P type doped Si channel



We have to include the doping N_a in the Poisson equation,

$$\frac{d^2\psi}{dx^2} = \frac{q}{\varepsilon_{Si}} \cdot \left(n_i \cdot e^{\frac{\psi - V}{U_T}} + N_a \right)$$

… but then, no analytical solution can be found ...

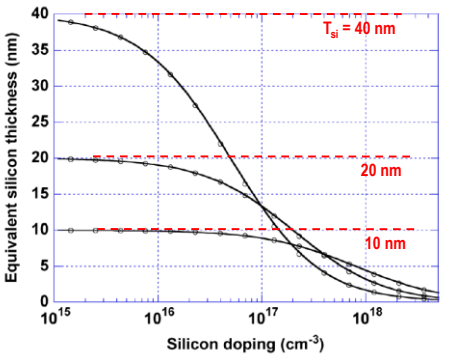
Highly Doped DG FETs

...but we can define an equivalent thickness, which is doping dependent, and use the undoped core model !

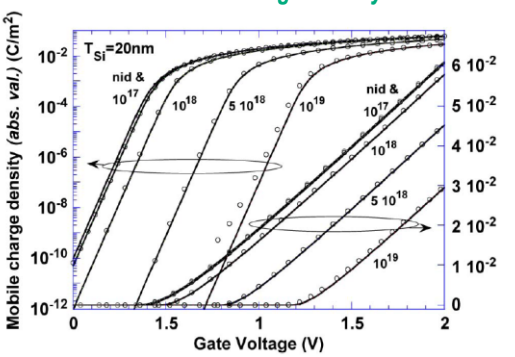
$$\frac{1}{\frac{T_{eq}}{2}} = \frac{1}{\int_{-T_{Si}/2}^{0} e^{\frac{X}{T_{Si}\cdot\varepsilon_{Si}\cdot U_T}\cdot x^2}\,dx} + \frac{X}{\varepsilon_{Si}\cdot U_T}$$

$$with \quad X = \frac{q\cdot N_a\cdot T_{Si}}{2}$$

Equivalent thickness



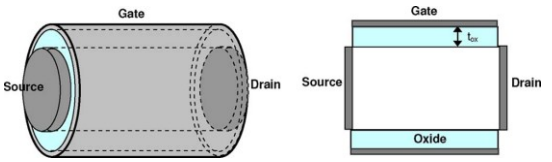
Mobile charge density



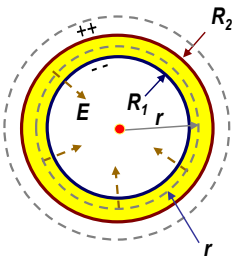
III- The Gate All Around FET.

The GAA FET

Surrounding the silicon by a gate is the most efficient geometry to control the electrostatic



Is $\varepsilon_{ox}/T_{si\ still}$ the capacitance per unit surface for a cylindical gate ?



(outside, E=0)

From symmetry, the electric field is uniform and radial. From Gauss theorem, the flux of the electric field accross the enclosing surfaces is propotionnal to the charge (per unit length) between these surface.

$$Q_G = 2\pi \cdot r \cdot E(r) \cdot \varepsilon_{ox}$$

The potential between the gate electrode and the potential @ R_1 is then:

$$V_{R2} - V_{R1} = \int_{R1}^{R2} E(r) \cdot dr = \frac{Q_G}{2\pi \cdot \varepsilon_{ox}} \int_{R1}^{R2} \frac{1}{r} \cdot dr = \frac{Q_G}{2\pi \cdot \varepsilon_{ox}} \cdot \ln\left(\frac{R_2}{R_1}\right)$$

The GAA FET

The capacitance per unit surface is defined as:

$$C = \frac{Q_G}{V_{R2} - V_{R1}} \cdot \frac{1}{2\pi \cdot R_1} = \frac{Q_G}{\frac{Q_G}{2\pi \cdot \varepsilon_{ox}} \cdot \ln\left(\frac{R_2}{R_1}\right)} \cdot \frac{1}{2\pi \cdot R_1} = \frac{\varepsilon_{ox}}{R_1 \cdot \ln\left(\frac{R_2}{R_1}\right)}$$

The radius and layer thickness are related each other by: $\begin{cases} T_{si} = 2 \cdot R_1 \\ T_{ox} = R_2 - R_1 \end{cases}$

Then the oxide capacitance per unit surface in a GAA device is:

$$C_{ox} = 2 \cdot \frac{\varepsilon_{ox}}{T_{si} \cdot \ln\left(\left(\frac{T_{si}}{2} + T_{ox}\right) \Big/ \frac{T_{si}}{2}\right)} = 2 \cdot \frac{\varepsilon_{ox}}{T_{si} \cdot \ln\left(1 + \frac{2 \cdot T_{ox}}{T_{si}}\right)}$$

If R_2-R_1 ~0, C_{OX} reverts to the planar gate capacitance:

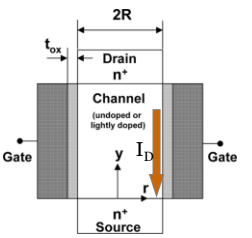
$$C_{ox} \approx 2 \cdot \frac{\varepsilon_{ox}}{T_{si} \cdot \frac{2 \cdot T_{ox}}{T_{si}}} \approx \frac{\varepsilon_{ox}}{T_{ox}}$$

The GAA FET

We consider undoped (lightly doped) cylindrical GAA-MOSFET with n-type S and D.

Symmetry of the system imposes cylindrical coordinates.
Poisson equation in combination with semiconductor statistics gives:

$$\frac{d^2\psi(r)}{d^2r}+\frac{1}{r}\cdot\frac{d\psi(r)}{dr}=\frac{q\cdot n_i}{\varepsilon_{si}}\cdot\exp\left(\frac{\psi(r)-V(r)}{U_T}\right)$$



And must satisfy the boundary conditions:

- Symmetry of E wrt the center of the cylinder: $\frac{d\psi(r=0)}{dr}=0$
- and at the surface: $\psi(R)=\psi_S$

This differential equation has an **exact** analytical solution given by:

$$\psi(r)=V+U_T\cdot\ln\left(\frac{-8\cdot B}{\left(1+B\cdot r^2\right)^2}\cdot\frac{U_T\cdot\varepsilon_{si}}{q\cdot n_i}\right)$$

The coefficient B is related to the surface potential from the limit condition.
At this stage of the derivation, its value remains unknown.

THE JOURNAL OF CHEMICAL PHYSICS VOLUME 20, NUMBER 11 NOVEMBER, 1952

On the Solution of the Poisson-Boltzmann Equation with Application to the Theory of Thermal Explosions

P. L. CHAMBRÉ
Department of Mathematics, University of California, Berkeley, California
(Received April 28, 1952)

The theory of thermal explosions originally proposed by Frank-Kamenetzky has been the subject of a number of investigations. The critical condition of inflammability requires the solution of the nonlinear Poisson-Boltzmann differential equation. For the case of a reaction vessel of cylindrical or spherical shape the solution was obtained by previous investigators by numerical integration of the equation. It is shown, however, in the following that the solutions can be obtained in terms of known functions.

$$\frac{d^2\theta}{dz^2}+\frac{k}{z}\frac{d\theta}{dz}=-\delta\exp(\theta),$$

The solution of this equation is

$$\theta=A-2\ln(Bz^2+1).$$

The GAA FET

This can also be written: $$\psi(r) = V + U_T \cdot \ln\left(-8 \cdot B \cdot \frac{U_T \cdot \varepsilon_{si}}{q \cdot n_i}\right) - 2 \cdot U_T \cdot \ln\left(\left|1 + B \cdot r^2\right|\right)$$

Potential @ film center: ~ ψ_0 for DG

The mobile charge density per unit surface, i.e. considered as a 'projection' on the cylinder cross section, i.e. half of the perimeter, is given by:

$$Q_{mob} = -\frac{2 \cdot \pi \cdot R \cdot C_{ox} \cdot (V_G - \Delta\phi - \psi_S)}{\pi \cdot R} = -2 \cdot C_{ox} \cdot (V_G - \Delta\phi - \psi_S)$$

From Gauss theorem, this charge should be related to the flux of the electric field enclosing the circular gate. Here Q_{mob} represents the charge density in the 'DG sense,

$$Q_{mob} = \frac{2 \cdot \pi \cdot R \cdot \varepsilon_{si} \cdot (-E_s)}{\pi \cdot R \cdot} = -2 \cdot \varepsilon_{si} \cdot \left.\frac{d\psi(r)}{dr}\right|_{r=R} \quad \text{and} \quad \frac{d\psi(r)}{dr} = -4 \cdot B \cdot U_T \cdot \frac{r}{1 + B \cdot r^2}$$

Then, for a given V_G-V we can show that we obtain an implicit relation in terms of B (though the parameter β):

$$\frac{V_G - \Delta\phi - V}{U_T} - \ln\left(\frac{\delta}{\delta \cdot R^2}\right) = \ln(1 - \beta) - \ln\left(\beta^2\right) + \eta \cdot \frac{1 - \beta}{\beta}$$

$\delta = \dfrac{q \cdot n_i}{\varepsilon_{si} \cdot U_T}$ *The SC parameter*

$\beta = 1 + B \cdot R^2$ *The unknown*

$\eta = \dfrac{4 \cdot \varepsilon_{si}}{C_{ox} \cdot R}$ *The structural parameter*

The GAA FET

Expressing β as a function of Q_{mob} leads to a charge based model.

$$\varepsilon_{si}\cdot\left.\frac{d\psi(r)}{dr}\right|_R = -\frac{Q_{mob}}{2} \quad \text{and} \quad \varepsilon_{si}\cdot\left.\frac{d\psi(r)}{dr}\right|_R = -4\cdot\varepsilon_{si}\cdot B\cdot U_T\cdot\frac{R}{1+B\cdot R^2} = Q_0\cdot\frac{1-\beta}{\beta} \quad \text{with} \quad Q_0 = \frac{4\cdot\varepsilon_{si}\cdot U_T}{R}$$

β can also be expressed in terms of the mobile charge density:

$$Q_{mob} = -2\cdot Q_0\cdot\frac{1-\beta}{\beta} \quad \Rightarrow \quad \beta = \frac{2\cdot Q_0}{2\cdot Q_0 - Q_{mob}}$$

We finally obtain the core charge based relationship for the GAA FET.

$$\frac{V_G-\Delta\phi-V}{U_T} - \ln\left(\frac{8}{\delta\cdot R^2}\right) = \frac{-Q_{mob}}{2\cdot C_{ox}\cdot U_T} + \ln\left(\frac{-Q_{mob}}{2\cdot Q_0}\right) + \ln\left(1-\frac{Q_{mob}}{2\cdot Q_0}\right)$$

Recalling the expression we obtained for symmetric DG MOSFET (normalized quantities)

$$v_G - v - v_{FB} + \ln\left(\frac{q_{\text{int}}}{2}\right) = 4\cdot q_g + \ln(q_g) + \ln\left(1+q_g\cdot\frac{C_{ox}}{C_{Si}}\right)$$

q_g: *for 1 gate*

Q_{mob}=-2 Q_g

$Q_{sp} = 4\cdot C_{ox}\cdot U_T$

$$v_G - v - v_{FB} + \ln\left(\frac{q_{\text{int}}}{2}\right) = 2\cdot\frac{-Q_{mob}}{Q_{sp}} + \ln\left(-\frac{Q_{mob}}{2\cdot Q_{sp}}\right) + \ln\left(1-Q_{mob}\cdot\frac{C_{ox}}{2\cdot C_{Si}\cdot Q_{sp}}\right)$$

DG

The GAA FET

As for the DG, we define a normalized charge factor for the GAA: $Q_{SP} = 4C_{ox}U_T$.

Defining the silicon capacitance of in GAA MOSFET by: $C_{Si} = \frac{\varepsilon_{si}}{R}$

we get $Q_0 = 4 \cdot U_T \cdot \frac{\varepsilon_{si}}{R} = 4 \cdot U_T \cdot C_{si}$

Already known

Then, the charge based relation for GAA MOSFET becomes:

$$v_G - \Delta\varphi - v - \ln\left(\frac{8}{\delta \cdot R^2}\right) = \frac{-2 \cdot Q_{mob}}{4 \cdot C_{ox} \cdot U_T} + \ln\left(\frac{-Q_{mob}}{4 \cdot C_{ox} \cdot U_T} \cdot \frac{2 \cdot C_{ox} \cdot U_T}{Q_0}\right) + \ln\left(1 - \frac{Q_{mob}}{4 \cdot C_{ox} \cdot U_T} \cdot \frac{2 \cdot C_{ox} \cdot U_T}{Q_0}\right)$$

Introduction of the normalization for charges

$$q_{mob} = Q_{mob} / 4 \cdot C_{ox} \cdot U_T$$

$$v_G - \Delta\varphi - v - \ln\left(\frac{8}{\delta \cdot R^2}\right) - \ln\left(\frac{C_{ox}}{2 \cdot C_{si}}\right) = -2 \cdot q_{mob} + \ln(-q_{mob}) + \ln\left(1 - q_{mob}\frac{C_{ox}}{2 \cdot C_{si}}\right)$$

$$\boxed{v_G - \Delta\varphi - v + \ln\left(\frac{q_{\text{int}}}{2}\right) = -2 \cdot q_{mob} + \ln(-q_{mob}) + \ln\left(1 - q_{mob}\frac{C_{ox}}{2 \cdot C_{si}}\right)}$$

$$q_{\text{int}} = \frac{(\pi \cdot R^2) \cdot q \cdot n_i}{(\pi \cdot R) \cdot 4 \cdot C_{ox} \cdot U_T}$$

The corresponding normalized intrinsic carrier density is consistent

GAA and DG FETs share the same relationship between potentials and mobile charge densities

The GAA FET

Adopting the same procedure as for the DG MOSFET, and defining an effective width as $W_{eff}=\pi R$ (in a 'DG sense'), the current is:

$$I = -\mu \cdot W_{eff} \cdot Q_m(y) \cdot \frac{dV(y)}{dy}$$

It can be shown that a normalization factor for the current is:

$$I_{SP} = 4 \cdot \mu \cdot C_{ox} \cdot U_T^2 \cdot \frac{W_{eff}}{L} = 4 \cdot \mu \cdot C_{ox} \cdot U_T^2 \cdot \frac{\pi \cdot R}{L}$$

Leading to a normalized current:

$$i = \frac{I}{I_{SP}} = -q_m^2 + 2 \cdot q_m + 2 \cdot \frac{C_{Si}}{C_{ox}} \cdot \mathbf{ln}\left(1 - q_m \cdot \frac{C_{ox}}{2 \cdot C_{Si}}\right)\Bigg|_{q_m(S)}^{q_m(D)}$$

Strong similarities exist between DG and GAA MOSFETs that lead to **a unique expression of charge-voltage relationships** provided a correct definition of gate oxide capacitance, silicon capacitance and specific normalizing charge density are used.

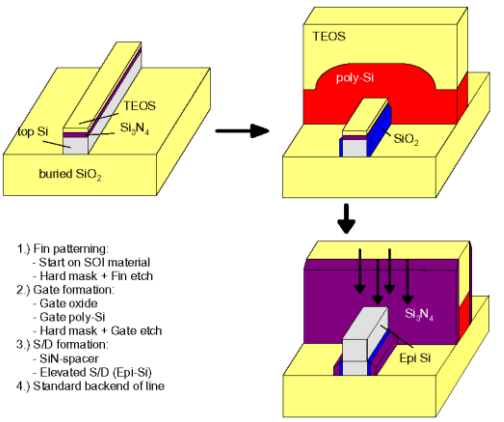
IV- Concept of equivalent parameters in arbitrary shape FETs.

FinFETS: structure and characteristics

- Double gate structures suffer from technological issues since both gates have to be aligned with a precision of the order of the nanometer.

  An interesting alternative comes from the FinFET structure.

- For instance, a FinFET is obtained by etching the active silicon layer of an SOI wafer, which defines the 'channel', followed by oxidation and contact formation.



1.) Fin patterning:
- Start on SOI material
- Hard mask + Fin etch

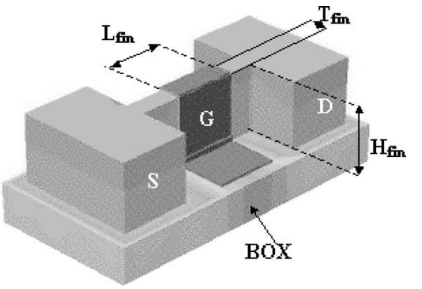
2.) Gate formation:
- Gate oxide
- Gate poly-Si
- Hard mask + Gate etch

3.) S/D formation:
- SiN-spacer
- Elevated S/D (Epi-Si)

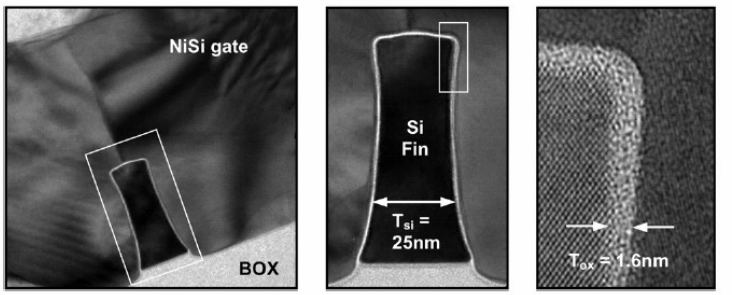
4.) Standard backend of line

FinFETS: structure and characteristics

One of the challenges in FinFET integration is the formation of 5–10 nm wide fins, required to fully benefit from the short channel control of multi-gate devices.
In addition, since ultra-thin Si films are needed to obtain good electrostatic control, the access resistance is very high in narrow fin devices and can limit the device efficiency

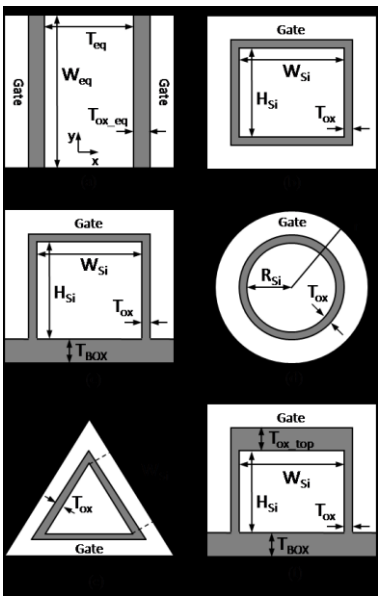


SEM and TEM images of a FinFET.



The Equivalent Channel Thickness

Generic multigate MOSFETs



All these topologies use a common gate electrode.

However, the gate insulator can be non homogeneous in a given structure.

The idea is to see if a representation of these 3D devices in terms of an equivalent planar DG MOSFET is still possible.

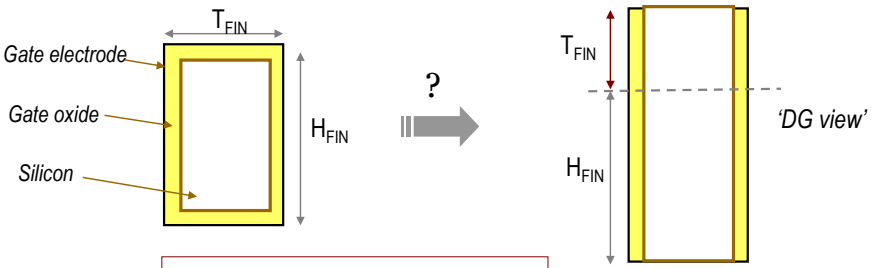
Unlike planar DG MOSFETs, the Boltzmann-Poisson equation should be solved in 2D:

$$\frac{d^2\Psi(x,y)}{dx^2} + \frac{d^2\Psi(x,y)}{dy^2} = -\frac{q \cdot n_i}{\varepsilon_{Si}} \cdot e^{\left(\frac{\Psi(x,y)-V_{ch}}{U_T}\right)}$$

Case of study: the QuadriGate

We notice that **above the threshold**, mobile charges will accumulate at the Si/SiO_2 interfaces: the device reverts to a quasi 1D system, we can simply add the contributions from each interface to calculate charges and currents.

The solution obtained for the long channel DG MOSFET could be used to estimate the charge density above the threshold for the rectangular FET.



In that case: $L_{EQ\_DG}=L_{FIN}$ and $W_{EQ\_DG}= H_{FIN} + T_{FIN}$.

However, **below the threshold**, the device should be treated in 2D since we expect the electric field to spread deeply in the silicon.

QuadriGate vs DG : the Equivalent Thickness

How can we adapt the symmetric DG model to simulate an ideal FinFET with a uniformly thick gate insulator (top and side gate capacitances equal) ?

• **Below threshold**, the mobile charge in the silicon is very small: *Q~0, and so* $\Psi_S \sim V_G - \Delta\phi$

• The solution is obtained from the Laplace equation and depends on the boundary conditions.

$\Delta\Psi = 0$

• For simplicity, we consider that the FinFET will be 'surrounded' by a uniform gate, i.e by a uniform oxide.

~ 'gate all around'

• Let us assume that the potential will be constant across the silicon. At least, this trivial solution satisfies Laplace equation

$\psi(x,y) = \psi_0$

This analysis can be used to link rectangular multigates and planar DG if we can satisfy that:
- **above the threshold**, we don't 'see' the volume.
- **below the threshold**, both devices are controlled by the charges in the volume.

QuadriGate vs DG : the Equivalent Thickness

If the rectangular FET can be **mapped on a DG FET**, we can write:

$$V_G - \Delta\phi - V_{ch} = \frac{Q_G}{C_{ox}} + U_T \cdot \ln\left(\frac{Q_G^2}{2\varepsilon_{Si} \cdot e \cdot U_T \cdot n_i} - C_1\right)$$

where C_1 represents an integration constant that **matters mainly below the threshold**.

Since this relation is known to give both surface and volume charge densities, the sensitive parameter should be C_1 which can be determined as follows.

In symmetric DG, C_1 was given from: $C_1 = -e^{\frac{\psi_0 - V_{CH}}{U_T}}$

In addition, if we suppose that below the threshold $\psi(x,y) = \psi_o$, carriers density/surface is:

$$Q_m \overset{Vg<<V_T}{=} -\frac{e \cdot n_i \cdot e^{\frac{\psi_0 - V}{U_T}} \cdot (H_{Fin} \cdot T_{Fin})}{(H_{Fin} + T_{Fin})}$$

We recognize that C_1 is related to the local mobile charge density below the threshold:

$$C_1 \overset{Vg<<V_T}{\approx} \frac{Q_m \cdot (H_{Fin} + T_{Fin})}{e \cdot n_i \cdot (H_{Fin} \cdot T_{Fin})}$$

QuadriGate vs DG : the Equivalent Thickness

The mobile charge density/surface is related to the charge on the 4 gates:

$$-Q_m = Q_G \cdot \frac{(2 \cdot H_{Fin} + 2 \cdot T_{Fin})}{(H_{Fin} + T_{Fin})} = 2 \cdot Q_G$$

For QuadriGate Fets, the integration constant C_1 becomes:

$$C_1 \overset{WI}{\approx} \frac{Q_m \cdot (H_{Fin} + T_{Fin})}{e \cdot n_i \cdot (H_{Fin} \cdot T_{Fin})} \cdot = \frac{-2 \cdot Q_G \cdot (H_{Fin} + T_{Fin})}{e \cdot n_i \cdot (H_{Fin} \cdot T_{Fin})} = \frac{-2 \cdot Q_G}{e \cdot n_i \cdot T_{EQ}} \quad \text{with} \quad T_{EQ} = \frac{H_{Fin} \cdot T_{Fin}}{H_{Fin} + T_{Fin}}$$

Then, formally the charge-potential relation is similar to that of the DG provided that an equivalent silicon thickness T_{EQ} is defined:

$$V_G - \Delta\phi - V = \frac{Q_G}{C_{ox}} + U_T \cdot \ln\left(\frac{Q_G^2}{2\varepsilon_{Si} \cdot e \cdot U_T \cdot n_i} + \frac{2 \cdot Q_G}{e \cdot n_i \cdot T_{EQ}}\right)$$

After applying the 'DG' normalization:

$$v_g - \Delta\phi - v + \ln\left(\frac{q_{\text{int}}^{eq}}{2}\right) = 4 \cdot q_g + \ln(q_g) + \ln\left(1 + q_g \cdot \frac{C_{ox}}{C_{EQ}}\right) \quad \text{Where} \quad C_{EQ} = \frac{\varepsilon_{Si}}{T_{EQ}}$$

With $q_{\text{int}}^{eq} = e \cdot n_i \cdot T_{EQ} / Q_{spec}$ and $Q_{spec} = 4 \cdot C_{ox} \cdot U_T$

QuadriGate vs DG : the Equivalent Thickness

As usual, the current is derived from the mobile charge density:

$$I = -\mu \cdot \overbrace{\frac{(H_{Fin} + T_{Fin})}{L_{Fin}}}^{W_{Eq}} \cdot \int_{V_s}^{V_D} Q_m \, dV = \mu \cdot \frac{2 \cdot (H_{Fin} + T_{Fin})}{L_{Fin}} \cdot \int_{V_s}^{V_D} Q_G \cdot dV$$

Then, we can define a specific current such that:

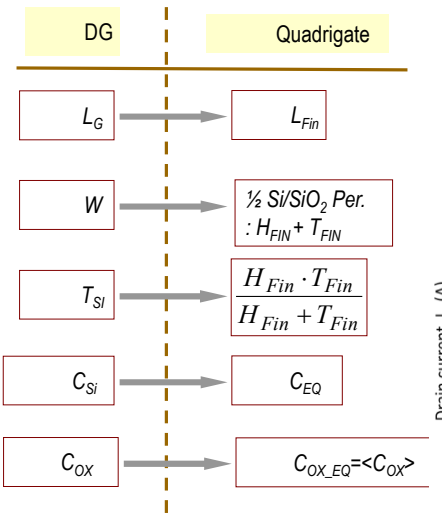
$$I_{spec} = 4 \cdot \mu \cdot C_{ox} \cdot U_T^2 \cdot \frac{W_{Eq}}{L_{Fin}}$$

The normalized current in FinFETs is then obtained through a DG like relationship:

$$i = -q_m^2 + 2 \cdot q_m + 2 \cdot \frac{C_{Eq}}{C_{ox}} \cdot \ln\left(1 - q_m \cdot \frac{C_{ox}}{2 \cdot C_{Eq}}\right)\Bigg|_{q_m(S)}^{q_m(D)}$$

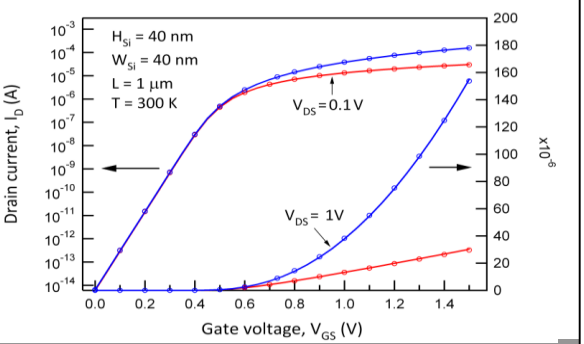
QuadriGate vs DG : the Equivalent Thickness

An equivalence between Finfets and Double Gate FETs exists via equivalent technological parameters.



Drain current in a long channel (1μm) QuadriGate FETS: good matching with the DG model, both below and above threshold, in linear and saturation.

$$T_{EQ} = \frac{H_{Fin} \cdot T_{Fin}}{H_{Fin} + T_{Fin}} = 20nm$$



Asymptotic expressions

How the 'Real' current will vary in FinFET with respect to symmetric DG for the same geometries ?

We suppose that: $T_{Fin} = \alpha \cdot H_{Fin}$ ➡ $T_{Eq} = \frac{1}{\alpha + 1} \cdot T_{Fin}$

For the FinFET, the specific current then becomes:

$$I_{spec\_FinFET} = 4 \cdot \mu \cdot C_{ox} \cdot U_T^2 \cdot \frac{H_{Fin} \cdot (1+\alpha)}{L_{Fin}} = I_{spec\_DG} \cdot (1+\alpha)$$

The specific charge Q_{SP} remains unaffected, whereas the normalized intrinsic charge density will be lowered in FinFET like structure.

- Assuming strong inversion operation and saturation, the normalized charge density at the source (V_S=0) and normalized current are given by:

$$4 \cdot q_g \approx v_g - \Delta\phi + \ln\left(\frac{q_{\text{int}}^{eq}}{2}\right)$$

no diff. between FinFET and DG

$$i \approx q_{mS}^2 \approx \left(-2q_g\right)^2 \approx \left(v_g - \Delta\phi\right)^2$$

Then, the current in FinFET in SI is higher than in DG: $I_{FinFET} \approx I_{DG} \cdot (1+\alpha)$

Asymptotic expressions

- Under weak inversion (and saturation), the normalized charge density at the source (V_S=0) is given by:

$$\ln(q_g) \approx v_g - \Delta\phi + \ln\left(\frac{q_{\text{int}}^{eq}}{2}\right)$$

Threshold voltage would be higher in FinFET like structure wrt to equivalent planar DG (?)

- The normalized current is then:

$$i \approx -2 \cdot q_{mS} \approx 4 \cdot e^{(v_g - \Delta\phi)} \cdot \frac{q_{\text{int}}^{eq}}{2}$$

- In strong inversion, the variation of q_{int} was ignored. However, this can no longer be assumed in weak inversion:

$$q_{\text{int}}^{eq} = e \cdot n_i \cdot T_{EQ} / Q_{spec} = q_{\text{int}}^{DG} \cdot \frac{1}{1+\alpha}$$

$$T_{Eq} = \frac{1}{\alpha + 1} \cdot T_{Fin}$$

- After de-normalization, and taking into account the correction of q_{int}, we find that the current in weak inversion should be almost the same in equivalent DG or FinFET devices:

$$I_{FinFET} \approx I_{DG} \cdot (1+\alpha) \cdot \frac{1}{(1+\alpha)} = I_{DG}$$

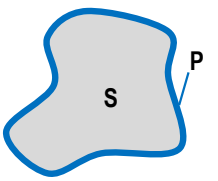
Generalization to arbitrary geometries

- We consider a device having a silicon channel of section S and perimeter P.
- We assume that the potential is uniform inside the device when biased below threshold.

Adopting the same DG concept, the integration constant is still obtained from the mobile charge density below threshold.

This gives a generalization of the equivalent thickness:

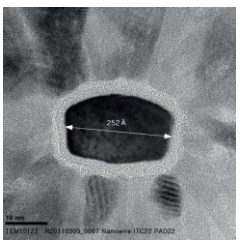
$$T_{EQ} = \frac{S}{P/2} = \frac{2 \cdot S}{P}$$



- But above threshold, the current should also scale with the perimeter.

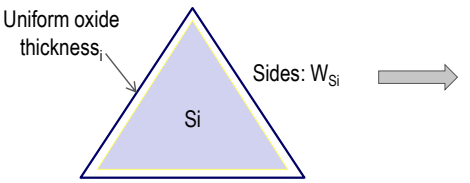
This defines an equivalent width: $W_{EQ} = \frac{P}{2}$

Typical NW: note the non-ideal circular shape … and gate oxide non uniformity



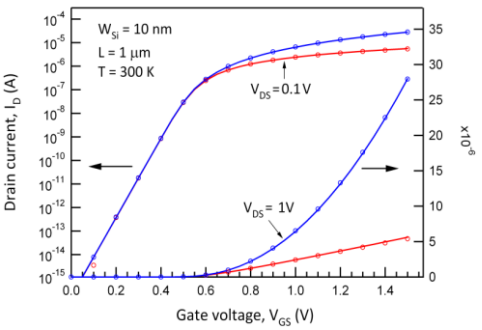
Generalization to arbitrary geometries

- Case of the equilateral triangle with 10 nm sidewalls.



$$T_{EQ} = \frac{W_{Si}}{2 \cdot \sqrt{3}} \approx 2.9nm$$

$$W_{EQ} = \frac{3 \cdot W_{Si}}{2} = 15nm$$



The approach is accurate even when devices exhibit sharp corners.

Generalization to arbitrary geometries

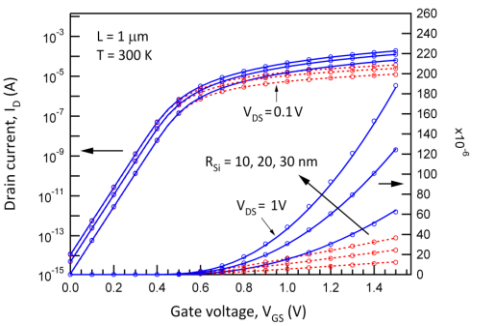
The 'worst' non-planar device is the cylindrical FET (GAA FET).

We can wonder if the approximate solution proposed so far still applies to the GAA FET

Applying the definition of the equivalent thickness concept to the GAA FET, we obtain:

$$T_{EQ} = \frac{2 \cdot \pi \cdot R^2}{2 \cdot \pi \cdot R} = R$$

Then,the equivalent thickness is <u>exactly</u> what the exact solution for the GAA FET gives through the definition of the silicon capacitance, i.e. $C_{si} = \varepsilon_{si}/R = \varepsilon_{si}/T_{EQ}$.



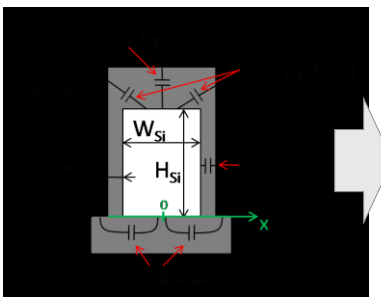
This confirms that :

- The charge based relation derived for the planar DG MOSFET is generic.
- The definition of T_{eq} is well sounded.

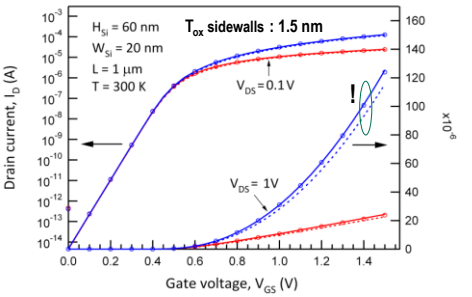
The Equivalent Gate Capacitance

The equivalent gate capacitance

So far, we analyzed specific structures where the gate insulator capacitance is constant all around the device. However, in real structures, this assumption does not hold anymore.
For instance, a FinFET has different oxide thicknesses and non-planar capacitances (fringing capacitances) reverting to a quite complex capacitive network.



Typical cross section of a FinFET



I_D of DG FinFET vs V_G with a 50 nm top oxide thickness. 3D simulations: symbols, DG model with T_{EQ}: dashed

Neglecting upper (and bottom) thick gate oxide generates some mistmatch regarding I_V characteristics: the drain current is underestimated above the threshold

The equivalent gate capacitance

At this point, we introduce 3 assumptions that will be justified *a posteriori*:

- We assume that the threshold voltages are the same whatever gate oxide interfaces.

- We consider that above the threshold, each channel is 'independent': the total current reverts to the sum of all Si/SiO_2 channel currents.

-Finally, we neglect the log term in the normalized current expression above threshold.

Then, the *normalized current* is $i^{SI} = -q_m^2 + 2 \cdot q_m$

Above threshold the normalized current only depends on the gate overdrive voltage (V_G-V_T).

Therefore, since V_G-V_T is assumed to have the same value at each interface, **above the threshold**, the total normalized current will be the sum of normalized current:

the total current can be expressed via the sum of specific currents for each channel:

$$I_D^{SI} = i^{SI} \cdot \left(I_{SP_{Lateral}} + I_{SP_{Top}} + I_{SP_{Bottom}} \right)$$

The equivalent gate capacitance

For a given interface, the specific current is : $I_{SP} = 4 \cdot \mu \cdot C_{ox_{interface}} \cdot U_T^2 \cdot \frac{W_{interface}/2}{L}$

According to the DG concept, each interface extension ($W_{interface}$) has to be divided by 2 since the equivalent DG representation should evidence a symmetry.

Therefore, the equivalent specific current will be given by:

$$I_{SP\_EQ} = 4 \cdot \mu \cdot U_T^2 \cdot \left(C_{ox} \cdot \frac{2 \cdot H_{si}/2}{L} + C_{ox\_top} \cdot \frac{W_{si}/2}{L} + C_{ox\_bottom} \cdot \frac{W_{si}/2}{L} \right)$$

Defining an equivalent channel width as: $W_{EQ} = H_{si} + W_{si}$

We can write $I_{SP} = 4 \cdot \mu \cdot C_{ox\_EQ} \cdot U_T^2 \cdot \frac{W_{EQ}}{L}$

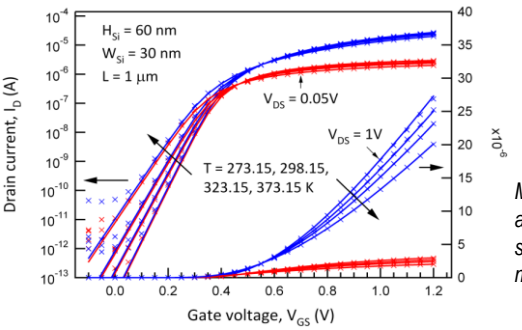
With the equivalent capacitance given by:

$$C_{ox\_EQ} = \frac{2 \cdot C_{ox} \cdot (H_{si}/2) + C_{ox\_top} \cdot (W_{si}/2) + C_{ox\_bottom} \cdot (W_{si}/2)}{H_{si} + W_{si}}$$

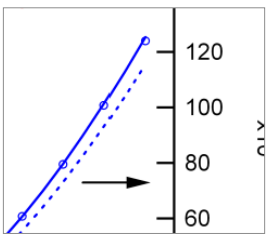
The equivalent gate capacitance

This definition for the equivalent gate capacitance can be generalized to any kind of multigate FET provided a single gate electrode is used (not independent gates)

From its definition, the equivalent capacitance can also be understood as the mean value of the gate capacitance per unit surface in the DG sense, i.e. considering that each gate electrode is turned into a 'symmetric' DG picture.



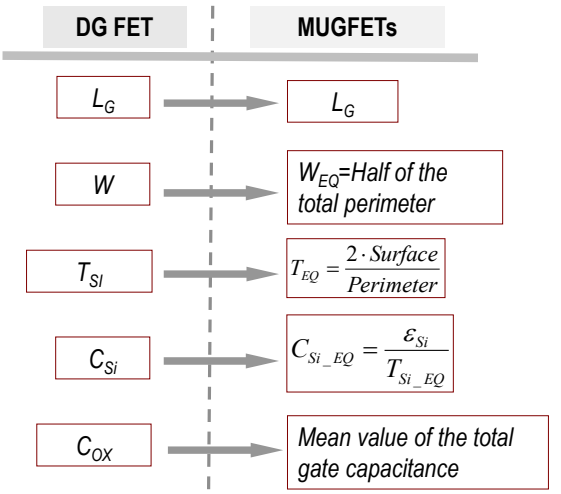
Magnification of I-V characteristics: Full and dotted lines: model with and without the equivalent capacitance



Model vs measurements: TG MOSFET as a function of gate voltage in linear and saturation regimes (Intel). measurements: crosses, model: lines.

MUGFETS and DG FET equivalences

The charge based model for the DG FET is therefore very generic. It can be used for almost whatever the geometry provided some transformation rules of technological parameters are done. It also inherits from powerful normalization concepts.

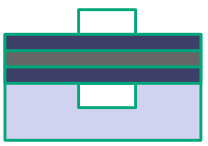


V. The Junctionless FETs

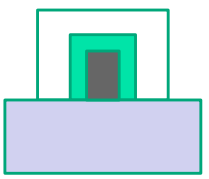
The ‘Simplest’ FET

Introduction to Junctionless FETs

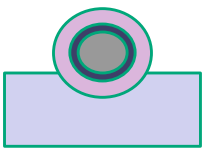
- Multigate FETs with undoped channels are interesting architectures because:
  - Better control of the channel with 'many' gates.
  - Almost ideal subthreshold slope (60mV/Dec).
  - Better immunity to random dopant fluctuations.
  - Ultra-sharp source-drain junctions suppressed



Double Gate

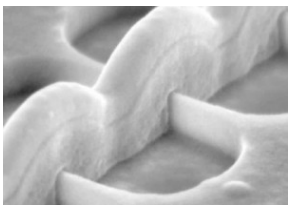


FinFET



Nanowire

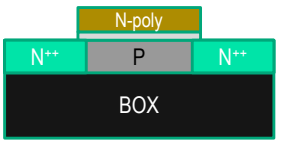
TSMC FinFET



Introduction to Junctionless FETs

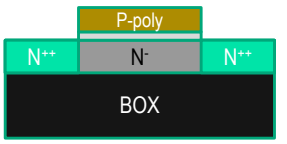
Combination of channel and source/drain doping profiles define three kinds of FET devices.

Inversion mode FET



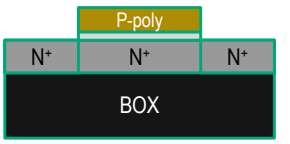
Inverted n-channel.
Interface, Minority carriers

Accumulation mode FET



Accumulated n-channel.
Interface, Majority carriers

JL FET



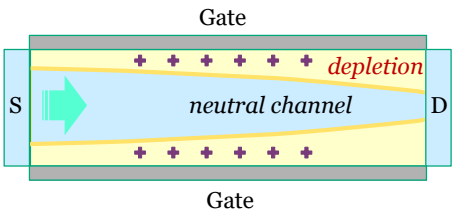
Accumulated/Depleted n-channel.
Volume, Majority carriers

In contrast to the accumulation mode FET, depletion is a key feature in JL FET operation.

Introduction to Junctionless FETs

- The device is a gated resistor with a heavily and uniformly doped (~ 10^{19} cm^{-3}) silicon channel.
- Channel switches-off gradually upon negative voltage applied to the gate (for n-type channel).
- Typical channel thicknesses: 10 to some nm.
- Architectures: planar Multigate and nanowires.

N-type doped (symmetric) double gate JL FET



Introduction to Junctionless FETs

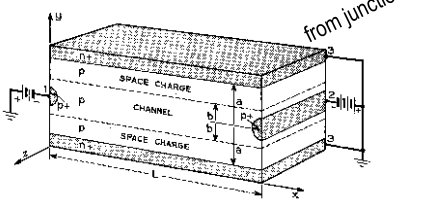
- Back in 1952 … and then was the JFET

  a Junctionless FET .. with Junction Gates

Proof of concept

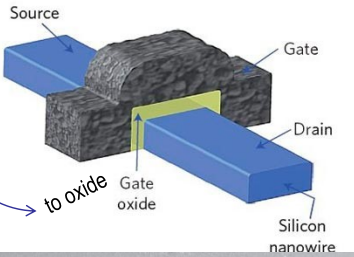
- Operation of a JLFET is based upon depletion of a highly doped semiconductor channel.
- No junction is needed since the current is carried by majority carriers in the volume.
- This contrasts with inversion mode FETs where minority carriers, supplied from source and drain junctions, are controlling the current.

….'A UNIPOLAR FIELD-EFFECT TRANSISTOR'
by SHOCKLEY W., in *PROCEEDINGS. OF THE INSTITUTE OF RADIO ENGINEERS - 1952*

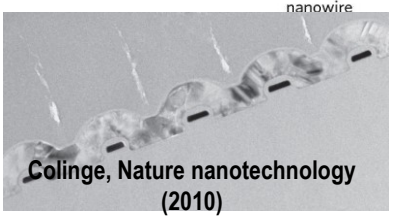


from junctions

to oxide



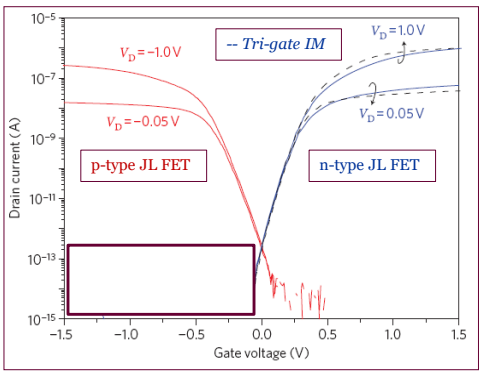
…preliminary concept also used 2 gates !

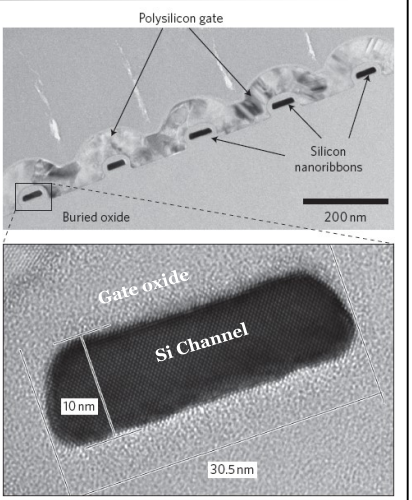


Colinge, Nature nanotechnology (2010)

Proof of concept

J.-P. Colinge et al., *Nanowire transistors without junctions,* Nature Nanotech., vol. 5, n° 3, pp. 225-229, 2010.

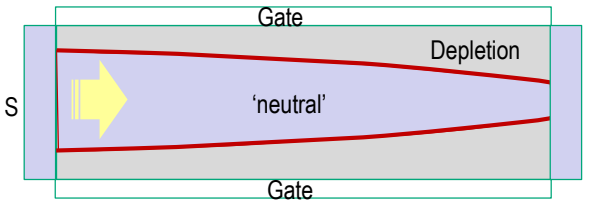




Performances in line with IM trigate FETs, but easier integration.

How JL DG FET works

'Uniform Doping' along Source-Channel-Drain: Junction-Less channel

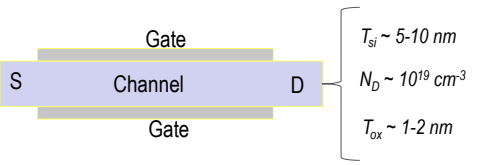


The current flows in the middle of the channel, not at the interfaces.

- In inversion mode FETs, what bothers is the depletion region, and its accurate evaluation is of 'little importance'…

- In junction-less FETs, what matters is the depletion region, which has to be calculate with care !

We consider a symmetric Double Gate structure, with a uniformly doped channel (N_D), and no junction.

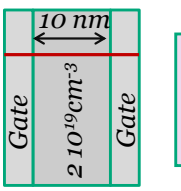


Pros and Cons of junctionless FET

Carrier mobility in JLFETs

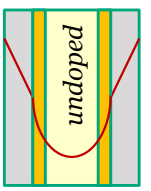
- Bulk conduction of JL FET is expected to improve mobility with respect to 'surface' conduction in IM FET:
  - In IM FETs, the normal electric field can be very large.
  - In JL FET at flat band, the electric field is negligible
    - improvement is then expected.

Double gate JL FET @ Flat Band



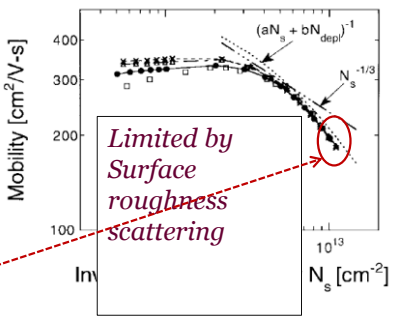
$E=0V/m$
$e\sim10^{13}cm^{-2}$

Inversion Mode DG FET



$E=10^{7}V/m$
$e\sim10^{13}cm^{-2}$

Scaled silicon MOSFET



D. Vasileska, TED Vol. 44, n. 4, p. 577 (1997)

Carrier mobility in JLFETs

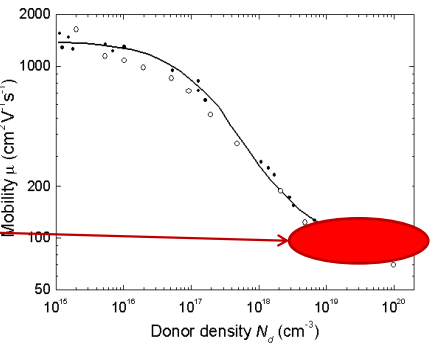
- But high doping can limit carrier mobility in JL FETs: impurity scattering dominates in JL FET.
- This mobility degradation is still an issue to overcome for these devices.

Jacoboni C. et al.
Solid State Electron.
20, 2, p. 77 (1977).

JLFET 10nm, 2 $10^{19}cm^{-3}$:

- μ *JL FET ~ 100 cm²/Vs*
- μ *DG FET ~ 200 cm²/Vs*

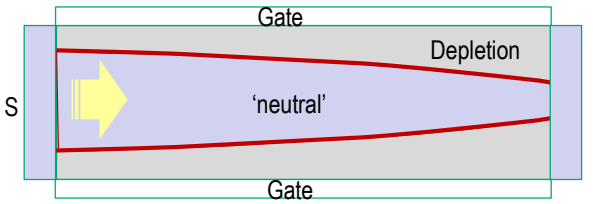
...but will not degrade any more while increasing doping.



Modelling the JL FET

How JL DG FET works

'Uniform Doping' along Source-Channel-Drain: Junction-Less channel



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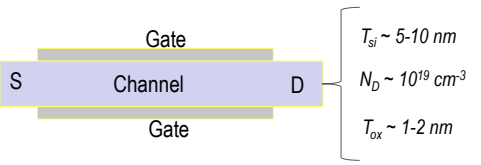


Is JL DG FET a simple resistance ?

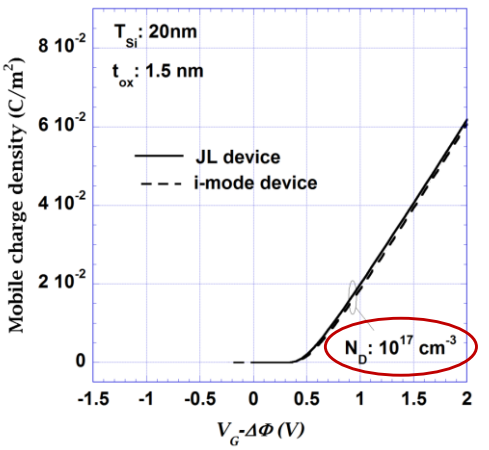
- In the simplest picture, in JL FETs the current flows uniformly across the channel thickness.
- This is indeed valid at flat band and low V_{DS} .
  - Then, the conductance scales with the doping and section of the channel.

$$\sigma \overset{\substack{V_{DS}\sim 0 \\ Flat\_Band}}{=} q \cdot \mu \cdot \frac{W}{L} \cdot N_D \cdot T_{Si}$$

  - … but this is not correct as soon as V_{DS} increases.

JL DG FET versus Inversion Mode FETs

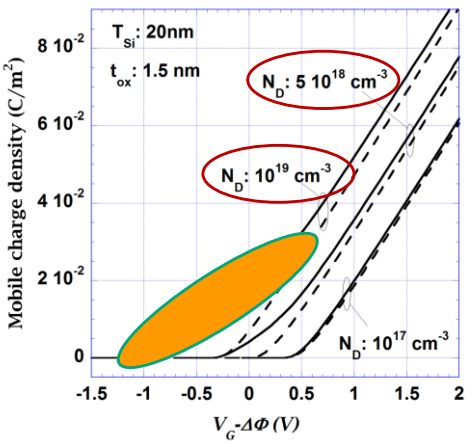
- Can the Double Gate JL FET be modeled as a an inversion mode DG FET ?
  - Do we really need for a new model ?



It seems that the charge density in a DG JL FET can be obtained from a DG MOSFET model for relatively low doped channels.

JL DG FET versus Inversion Mode FETs

Increasing the channel doping distorts noticeably the charge-voltage dependence.



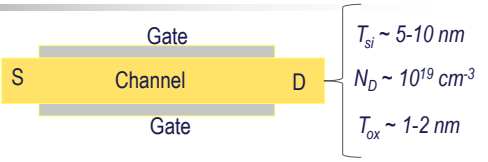
In fact, there is no way to simulate a DG JL FET with a regular DG FET model.

Model roots are different.

Charges versus Potentials in JL DG FET

Double Gate structure, with a uniformly doped channel (N_D).

No p-n junction.



Regarding modeling aspects, having a doped channel makes a big difference with respect to undoped junction based FETs.

An additional term due to ionized dopants (N_D) must be taken into account in Poisson equation:

$$\frac{d^2\psi}{dx^2} = \frac{q}{\varepsilon_{Si}}\left(n_i \cdot e^{\frac{\psi - V}{U_T}} - N_D \right)$$

Depending on the potentials, the semiconductor can be:

- Depleted: $\psi - V << U_T$
- Neutral: $\psi - V = U_T \, Ln(N_D/n_i)$
- Accumulated: $\psi - V >> U_T$

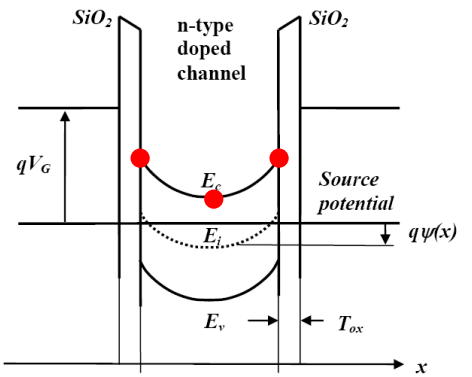
Charges versus Potentials in JL DG FET

- Poisson-Boltzmann equation has no analytical solution.

$$\frac{\partial^2 \Psi}{\partial x^2} = -\frac{q}{\varepsilon_{Si}}\left(-n_i \cdot e^{\frac{\psi(x)-V}{U_T}} + N_D\right)$$

V is the channel potential , i.e the Fermi potential for electrons

- Approximate solution based on finite difference in 3 points inside the channel.



Charges vs Voltages in JL DG FET

A crude approximation would neglect mobile charge density, i.e. electrons in our case: full depletion approximation.

Not acceptable since the charge factor in the Poisson equation can vary from *$-N_D$ (full depletion)* up to several N_D (accumulation).

Integrating once from the center to the semiconductor interface, the surface electric field becomes a function of the center and surface potentials *(E_{centre}=0 by symmetry):*

$$E_S^2 = \left(\frac{Q_{SC}}{2\cdot\varepsilon_{Si}}\right)^2 = \frac{2\cdot q\cdot n_i\cdot U_T}{\varepsilon_{Si}}\left[\left(e^{\frac{\psi_S - V}{U_T}} - e^{\frac{\psi_0 - V}{U_T}}\right) - \frac{N_D}{n_i}\frac{(\psi_S - \psi_0)}{U_T}\right]$$

Knowing the surface potential is not enough to evaluate the charge in the silicon.

This also depends on the potential at the center.

These variables can be linked by expressing the second derivative in terms of finite differences, where the discretization points are the center and the interfaces:

$$\left.\frac{d^2\psi}{dx^2}\right|_{x=0} = \left(\frac{\psi\left(\frac{-T_{SC}}{2}\right) - \psi(0)}{\frac{T_{SC}}{2}} - \frac{\psi(0) - \psi\left(\frac{T_{SC}}{2}\right)}{\frac{T_{SC}}{2}}\right)\cdot\frac{1}{\frac{T_{SC}}{2}} \quad \textit{with} \quad \left.\frac{d\psi}{dx}\right|_{x=0} = \frac{\psi\left(\frac{T_{SC}}{2}\right) - \psi\left(\frac{-T_{SC}}{2}\right)}{T_{SC}} = 0$$

Charges vs Voltages in JL DG FET

We obtain

$$\left(\psi_S - \psi_0\right) = \frac{q \cdot T_{SC}^{\ 2}}{8 \cdot \varepsilon_{Si}} \left(n_i \cdot e^{\frac{\psi_0 - V}{U_T}} - N_D \right)$$

❑ This relation links the center and surface potentials.
Note the dependence on silicon thickness and doping density.

The total charge is made of the mobile Q_m and fixed charges, and must be balanced by the charge on the two gates:

$$\begin{cases} Q_{SC} = Q_m + qN_D T_{SC} \\ \left(V_G - \Delta\phi - \psi_S\right) \cdot 2 \cdot C_{ox} = -Q_{SC} \end{cases}$$

After a detailed analysis, we can show that the solution can be split in two modes of operation:

❑ Depletion or accumulation mode.

Charges vs Voltages in JL DG FET

❑ In depletion:

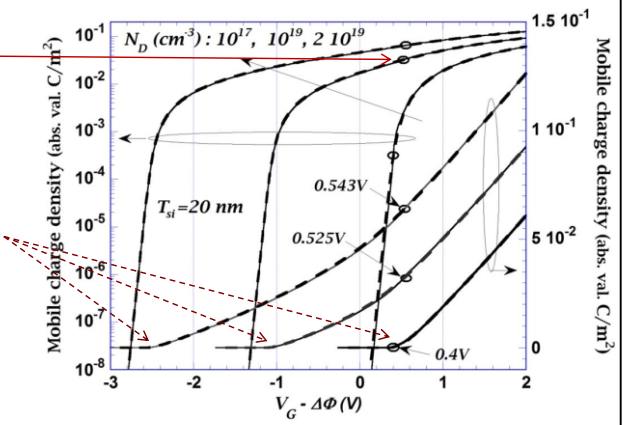
$$V_G - V - \Delta\phi - U_T \cdot \ln\left(\frac{N_D}{n_i}\right) \overset{dep}{\approx} -\frac{Q_{SC}^{\,2}}{8 \cdot q \cdot N_D \cdot \varepsilon_{Si}} - \frac{Q_{SC}}{2 \cdot C_{ox}} + U_T \cdot \ln\left(1 - \left(\frac{Q_{SC}}{q \cdot N_D \cdot T_{SC}}\right)^2\right)$$

❑ In accumulation

$$V_G - V - \Delta\phi - U_T \cdot \ln\left(\frac{N_D}{n_i}\right) \overset{acc}{\approx} \frac{-Q_{SC}}{2 \cdot C_{ox}} + U_T \cdot \ln\left(1 + \frac{Q_{SC}^{\,2}}{8 \cdot q \cdot N_D \cdot \varepsilon_{Si} \cdot U_T}\right)$$

❑ Circles indicate when the silicon is neutral in all the volume. This is the flat band condition

❑ Eventhough the threshold voltage does not have the same meaning as in junction based FETs, we can define it as the *x-intersection* of the mobile charge when extrapolated from the quasi linear section, before subthreshold.



Threshold voltage considerations

In accumulation mode:

The device looks like a 'regular' inversion mode DG FET… A threshold voltage concept is then possible.

Neglecting the *Ln* term and setting $Q_{SC} = qN_D T_{SC}$, we obtain:

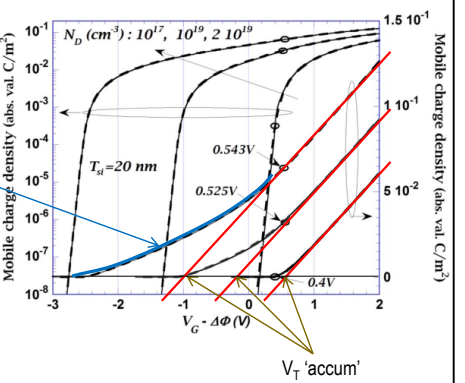
$$V_T = \Delta\phi + U_T \cdot \ln\left(\frac{N_D}{n_i}\right) - q \cdot N_D \cdot T_{SC} \cdot \left(\frac{1}{2 \cdot C_{ox}} + \frac{1}{8 \cdot C_{SC}}\right)$$

In depletion mode:

The 'distortion' in the Q-V characteristic comes from the quadratic term of the charge density.

This contribution is missing in the inversion mode DG FET !

A threshold voltage concept becomes meaningless: $C_{ox} \cdot (V_G - V_T) \neq Q_{SC}$



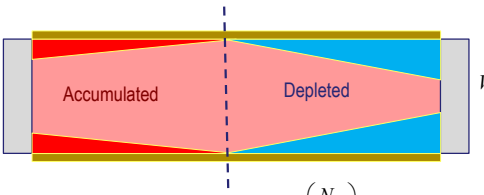
Note that for relatively low doped channels, the devices will mainly operate in accumulation

The Hydrid channel

Under specific bias, the channel can combine accumulation close to the source with depletion near the drain: this corresponds to the **hybrid channel case.**

$$V_G > \Delta\phi + U_T \cdot ln\left(\frac{N_D}{n_i}\right)$$

Source is the reference



$$V_D > V_G - \Delta\phi - U_T \cdot ln\left(\frac{N_D}{n_i}\right)$$

$$V_G - V_{tran} = \Delta\phi + U_T \cdot ln\left(\frac{N_D}{n_i}\right)$$

In hybrid operation, the current can be split in some '**below flat band**' and '**above flat band**' operation:

$$I = \frac{W}{L} \cdot \mu \cdot \int_S^D (qN_D T_{sc} - Q_{SC}) \cdot dV =$$
$$\frac{W}{L} \cdot \mu \cdot \int_S^{FB} (qN_D T_{sc} - Q_{SC}) \cdot dV + \frac{W}{L} \cdot \mu \cdot \int_{FB}^D (qN_D T_{sc} - Q_{SC}) \cdot dV$$

Where each integral has a well defined value.

Derivation of the current in JL FET

The current is given by:

'Bulk' Current

$$I = -W \cdot \mu \cdot Q_m \cdot \frac{dV}{dx} \quad \Rightarrow \quad I = \frac{W}{L} \cdot \mu \cdot \int_S^D \left(qN_D T_{sc} - Q_{SC}\right) \cdot dV = \frac{W}{L} \cdot \mu \cdot q \cdot N_D \cdot T_{sc} \cdot V_{DS} - \frac{W}{L} \cdot \mu \cdot \int_S^D Q_{SC} \cdot dV$$

Depending on the operation mode, the current will take different expressions:

The current in depletion

$$\int_S^D Q_{SC} \cdot dV \Big|_{dep} = \frac{1}{12 \cdot q \cdot N_D \cdot \varepsilon_S \cdot U_T} \cdot Q_{SC}^3 \Big|_S^D + \frac{1}{4 \cdot C_{OX}} \cdot Q_{SC}^2 \Big|_S^D - 2 \cdot U_T \cdot Q_{SC} \Big|_S^D + U_T \cdot \left(q \cdot N_D \cdot T_{sc}\right) \cdot \left(\ln\left(1 + \frac{Q_{SC}}{q \cdot N_D \cdot T_{sc}}\right) - \ln\left(1 - \frac{Q_{SC}}{q \cdot N_D \cdot T_{sc}}\right) \right) \Big|_S^D$$

The current in accumulation

$$\int_S^D Q_{SC} \cdot dV \Big|_{acc} = \frac{1}{4 \cdot C_{OX}} \cdot Q_{SC}^2 \Big|_S^D - 2 \cdot U_T \cdot \left(Q_{SC} - \sqrt{8 \cdot q \cdot N_D \cdot \varepsilon_{Si} \cdot U_T} \cdot arctg\left(\frac{Q_{SC}}{\sqrt{8 \cdot q \cdot N_D \cdot \varepsilon_{Si} \cdot U_T}} \right) \right) \Big|_S^D$$

The gate transconductance in JL DG FET

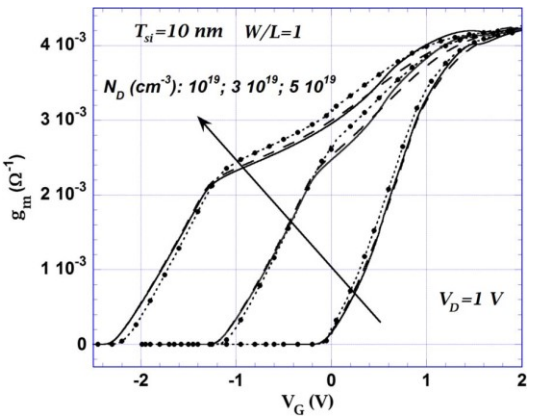
In depletion, there are two slopes.

This behavior is specific to JL FET and has no counterpart in regular DG FETs.

From the charge-potential relationship in depletion, in addition to the linear term, there is a **quadratic term that generates this non-linearity.**

Therefore, also the gate transconductance will de different from the junction based DG figure.

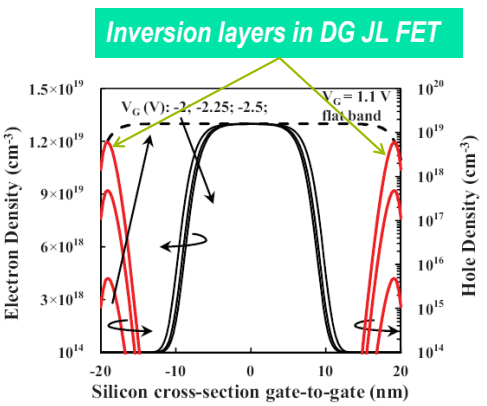
Those dissimilarities are less prominent when the doping or silicon thickness are decreased

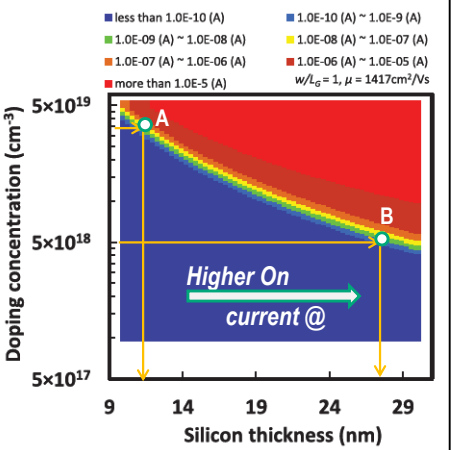


Design issues of JL DG FET

- To increase performance (on-current), higher doping and thicker channels are needed . But then the depletion region can reach an asymptote, whatever the magnitude of the gate voltage : the FET cannot be switched off .

- The reason for a lower limit to I_{OFF} is the occur channels)



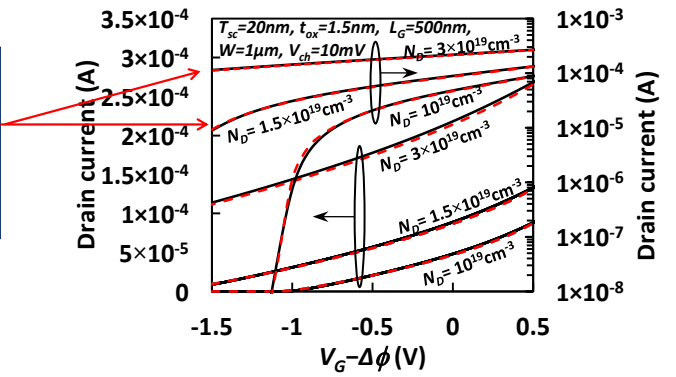


$$\frac{\sigma_A}{\sigma_B} \approx \frac{7 \cdot 10^{18} \cdot 11}{5 \cdot 10^{18} \cdot 27} = 0.57$$

Design issues of JL DG FET

- How to increase devices performances (current) ?
  - Higher doping and/or thicker channels.
- But there is an inherent limit to the depletion charge.
  - Maximum depletion is independent of the gate voltage
  - Instrinsic lower limit to off-state current.

Switch-off is not feasible for these [*Doping-Thickness*] combinations



Real case:
Assymetric operation of JL FETs

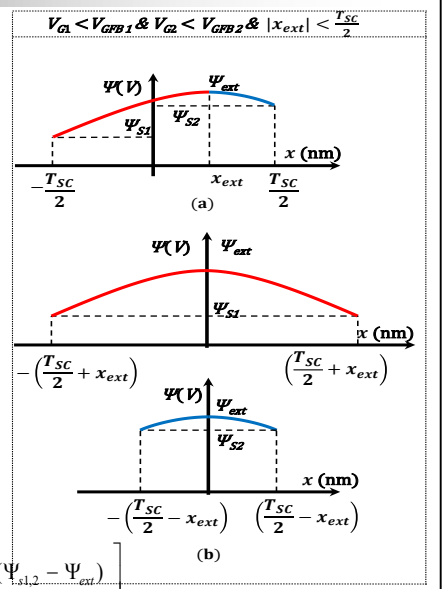
Asymetric operation in JL DG FET

- Asymetric operation can take place when gate oxides and/or gate voltages are different.

- From a potential point of view, when both gate voltages are below the flat bands, the potential is no longer symmetric.
But there is an extremum x_{ext} which (in our conditions) lies inside between the gates.

- The electric fields at each channel interface are obtained by integrating Poisson-Boltzmann equation from $-T_{sc}/2$ to x_{ext}, then from x_{ext} to $T_{sc}/2$ (the electric field vanishes at x_{ext}):

$$E_{s1,2}^2 = \frac{2qn_iU_T}{\varepsilon_{si}}\left[\exp\left(\frac{\Psi_{s1,2}-V_{ch}}{U_T}\right)-\exp\left(\frac{\Psi_{ext}-V_{ch}}{U_T}\right)-\frac{N_D}{n_iU_T}\left(\Psi_{s1,2}-\Psi_{ext}\right)\right]$$



Asymetric operation in JL DG FET

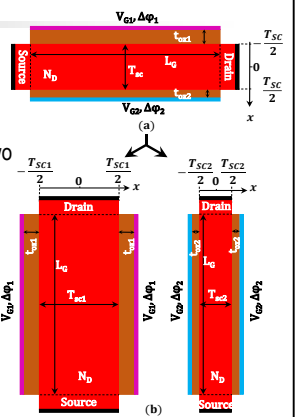
In case of symmetric operation, we know that once the gate potential and the silicon thickness are known, the charge density follows from the symmetric charge based model

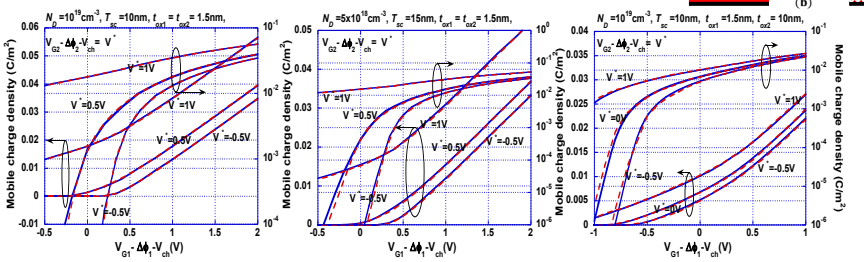
It comes out that this relation can also be considered as the solutions for two virtual symmetric devices (*DG*1 and *DG*2) having silicon thicknesses :

$$T_{SC1} = T_{SC} + 2x_{ext} \qquad T_{SC2} = T_{SC} - 2x_{ext}$$

Combining these 'virtual symmetric' devices with new physical parameters (T_{sc}, t_{ox} and $\Delta\varphi$) gives back the same surface potentials and charge densities as for the asymmetric case.

- Additional problem: find x_{ext} for given V_{g1} and V_{g2} !





The nanowire FET (JL)

Generalization to nanowire JL FETs

❑ Defining equivalent parameters, it is possible to simulate charges and current in JL nanowire using the relationships developed for the DG counterpart.

In cylindrical coordinates, the Poisson – Boltzmann equation writes:

$$\frac{d^2\psi}{dr^2} + \frac{1}{r}\cdot\frac{d\psi}{dr} = \frac{q}{\varepsilon_S}\left(n_i \cdot e^{\frac{\psi - V}{U_T}} - N_D\right)$$

No analytical solution…

❑ The idea is to perform a trapezoidal integration between the center and the surface.

After manipulation, we obtain an approximate relation between the surface field and the integral of the charge density in the semiconductor:

$$\left(\frac{1}{2}\cdot E_S^2\right)^{NW} \approx \frac{q}{\varepsilon_S}\cdot\int_{\psi_0}^{\psi_S}\left(\frac{n_i}{2}\cdot e^{\frac{\psi - V}{U_T}} - \frac{N_D}{2}\right)\cdot d\psi$$

… but at the same time, integration of the Poisson – Boltzmann equation for the DG gives:

$$\left(\frac{1}{2}\cdot E_S^2\right)^{DG} \approx \frac{q}{\varepsilon_S}\cdot\int_{\psi_0}^{\psi_S}\left(n_i \cdot e^{\frac{\psi - V}{U_T}} - N_D\right)\cdot d\psi$$

Therefore, we obtain the same relationship as for the 'well known' DG **provided the channel doping and intrinsic carrier density are divided by 2.**

Modeling JL Nanowire FET

Simple equivalence exist between Double Gate and Nanowire JL FETs.

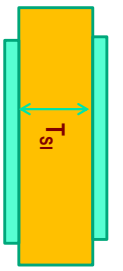
Nanowire JL FET

$$\frac{d^2\psi}{dr^2} + \frac{1}{r}\cdot\frac{d\psi}{dr} = \frac{q}{\varepsilon_{si}}\left(n_i \cdot e^{\frac{\psi(r)-V_{ch}}{U_T}} - N_D\right)$$

DG JL FET

$$\frac{d^2\psi}{dr^2} = \frac{q}{\varepsilon_{si}}\left(n_i \cdot e^{\frac{\psi(r)-V_{ch}}{U_T}} - N_D\right)$$

Physical param,	NW	Equivalent DG
Radius-Thickness	R	T_{si}=2R
Oxide thickness	T_{ox}	0.5 T_{ox}Ln(1+ 2 T_{ox}/T_{si})
Width	-	W=πR
Doping	N_D	N_D/2
Intrinsic carrier	n_i	n_i/2



Impact of surface traps

Traps in semiconductors

Nanowires and traps

Inclusion of surface traps in the nanowire model:

We assume an *n*-type semiconductor with a dopant density (N_D) and acceptor traps N_S located at the semiconductor/oxide interface for both geometries.

$$\varphi_s = \frac{qT_{sc}^{\ 2}}{8\varepsilon_{si}}\left(n_i \exp\left(\frac{\varphi_0 - V}{U_T}\right) - N_D\right) + \varphi_0$$

$$E_s = \frac{Q_{sc}}{2\varepsilon_{si}} = \left[\frac{2qn_iU_T}{\varepsilon_{si}}\left[\left(e^{\frac{\varphi_s - V}{U_T}} - e^{\frac{\varphi_0 - V}{U_T}}\right) - \frac{N_D}{n_i}\left(\frac{\varphi_s - \varphi_0}{U_T}\right)\right]\right]^{1/2}$$

$$Q_{SS}^{-} = -q\frac{N_s}{1+\exp\left(\frac{E_{t-i}}{kT}\right)\exp\left(-\frac{\varphi_s + V}{U_T}\right)}$$

$$Q_{sc} = -2Q_G = -2C_{ox}\left(V_G^* + \frac{Q_{SS}^{-}}{C_{ox}} - \varphi_s\right)$$

Nanowires and traps

Nanowires are often used as biosensors.

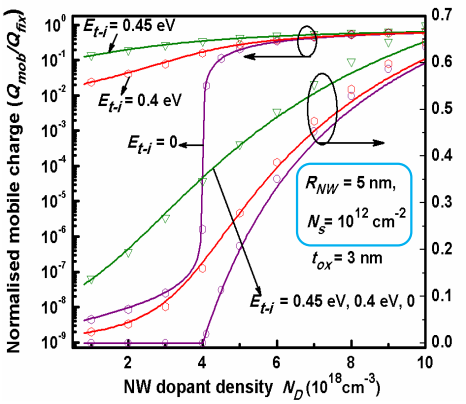
The techniques used to fabricate these devices are subject to introduce-generate defects at the semiconductor-gate insulator interface.

These defects may act as deep traps and impact the electrical properties of the sensor

In addition, these nanowires are in contact with liquids (electrolytes, liquids with bio molecules) , which is a kind of 'contamination' with in regard to the drastic control of impurities in a clean room.

Ungated JL NW - Self depletion

For a given doping level, decreasing the energy of the traps with respect to the intrinsic level 'will' remove the free carriers from the channel.



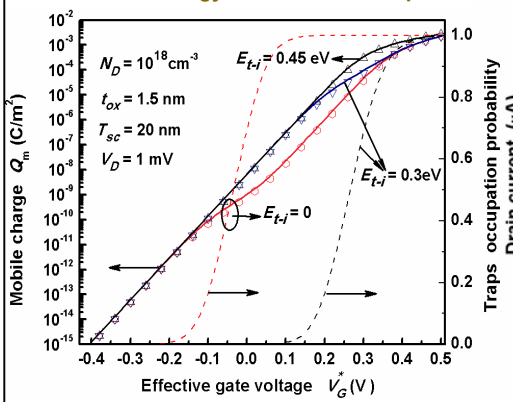
Even without a gate electrode, the electrical conductivity of the device will be highly influenced by a relatively low density of defects on the semiconductor-insulator interface.

Nanowires and traps

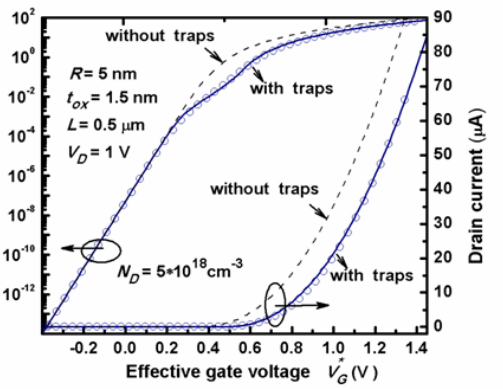
Gated JL NW

Traps are changing V_T 'dynamically'.
Depending on their energy, they will affect sub- and above threshold characteristics.

Discrete energy distribution of traps



Continous energy distribution of traps



VI - Ballistic transport in nano-transistors

What is ballistic transport ?

Transport in ultra short channels

- When the transistor gate length is about 10 nanometers, collisions of free carriers are expected to decrease: a quasi-ballistic regime is expected. Then, some questions arise:

- Is the mobility concept still valid ?

- Can drift-diffusion still be used for transport ?

- In ultra short devices, these concepts are becoming less valid , a 'new' theory is required.

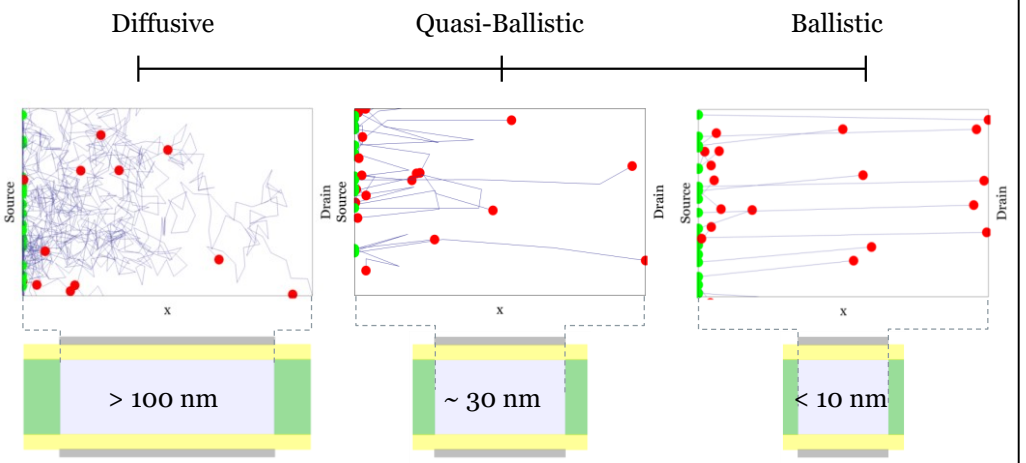
- A simple approach considers that the virtual source of the device is the strategic point for charge injection inside the channel from thermionic effect.

- The emission takes place primarily at the virtual source which is the point of maximum potential energy along the channel.

- At this point, the current consists of **a flux injected from the source** into the channel (with an average velocity) and eventually a flux of back-scattered carriers.

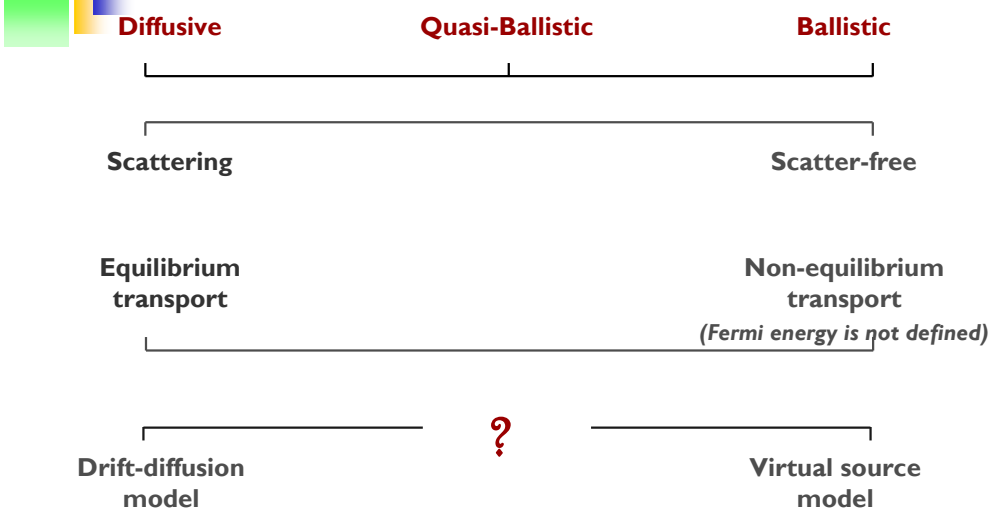
The picture inside the channel

- A simplistic view of charge transport inside the MOSFET channel:



Electrons travel across the channel **without scattering** in a ballistic MOSFET

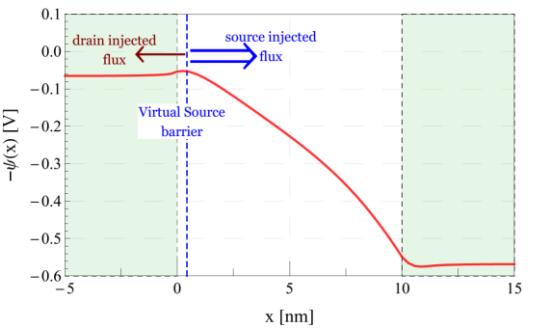
What changes between Diffusive and Ballistic ?



The virtual source.
Fluxes

The virtual source

- A simple approach considers that the virtual source of the device is the strategic point for charge injection inside the channel from thermionic effect.
- This emission takes place primarily at the virtual source which is the point of maximum potential energy along the channel.
- At this point, the current consists of a flux injected from the source into the channel (with an average velocity) and a flux of back-scattered carriers.



The virtual source barrier:

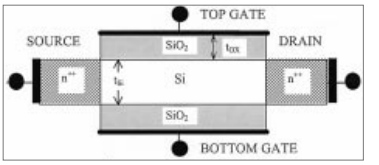
•The potential hump act as the effective source of carriers

•The gate voltage controls its height

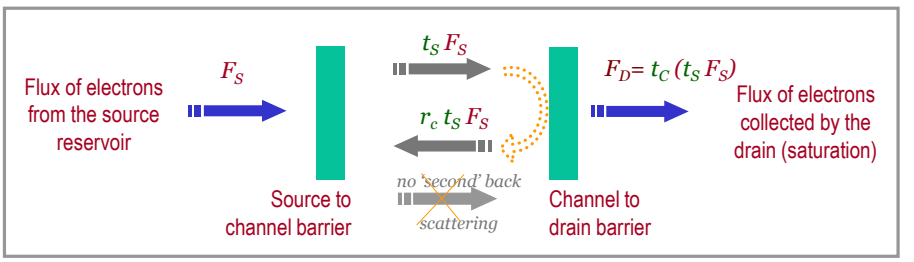
Scattering and fluxes

The flux concept was already introduced in the 60's by McKelvey.

The basic idea is to decompose the current into fluxes that travel in positive and negative directions



Scattering theory relates the steady-state current to transmission and reflection coefficients. This offers a new dimension to conventional transport models that are based on the net current. Through the concept of flux, we get closer to a microscopic description in terms of particles that build up a net current.



t_S , t_C : *transmission coefficients* $r_C = 1 - t_C$: *reflection coefficient @drain*

After entering into the drain, scattering in transverse modes will prevent electrons returning to the source.

Scattering and fluxes

Under equilibrium, the 'forward' and 'backward' oriented electron density at the source barrier must satisfy:

$$\begin{cases} n_{S+} \cdot v_{th} = F_S \cdot t_S \\ n_{S-} \cdot v_{th} = r_C \cdot F_S \cdot t_S \end{cases}$$

The electron density at the source barrier is then:

$$n_S = \frac{F_S \cdot t_S + r_C \cdot F_S \cdot t_S}{v_{th}} = \frac{F_S \cdot t_S (1 + r_C)}{v_{th}}$$

$$v_{th} \overset{2D\rightarrow}{=} \sqrt{\frac{2 \cdot kT}{\pi \cdot m_{trans}}}$$

Equilibrium implies that:

$$\langle v_+ \rangle = \langle v_- \rangle = v_{th}$$

Then, the flux entering at the source is expressed in terms of the electron density n_s obtained from equilibrium conditions (not valid at the drain), i.e. we can use MOS electrostatics:

$$n_S = \frac{F_D \cdot (1 + r_C)}{t_C \cdot v_{th}} = \frac{F_D \cdot (1 + r_C)}{(1 - r_C) \cdot v_{th}} = \frac{C_{ox}}{q} \cdot (V_{GS} - V_T)$$

Does not depend on V_{DS}

The drain current is then obtained as a function of the scattering parameter at the drain:

$$I_{Sat} = q \cdot F_D \cdot W = C_{ox} \cdot W \cdot V_{GT} \cdot \left[v_{th} \cdot \frac{1 - r_C}{1 + r_C} \right]$$

Apparent mean velocity of carriers:

$$\langle v \rangle = v_{th} \cdot \frac{1 - r_C}{1 + r_C}$$

The ballistic limit corresponds to $r_C = 0$. The maximum current density is then:

$$I_{Ballistic} = C_{ox} \cdot W \cdot v_{th} \cdot V_{GT}$$

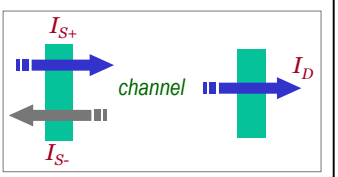
It represents the maximum current as given by the thermal velocity of the injected carriers at the source (then there is still some channel resistance)

Vth is the equilibrium unidirectional thermal velocity (i.e., the average velocity of carriers crossing the plane x=0 in the positive direction

Scattering and fluxes

…. A slightly different view….

Suppose that we know the transmission coefficient t.
A given drain current means that there is also a backward current flowing at the source.



Then, the absolute value of all kind of currents at the source must also include this contribution: I_S is the sum of I_D and I_{D_back} … and these global 'microscopic' currents revert to the sum of all 'thermal' currents, i.e. $n_S v_{th}$

$$\left.\begin{aligned} I_{S+} &= \frac{I_D}{t_C} \\ I_{S-} &= I_{S+} \cdot r_C = I_D \cdot \frac{r_C}{t_C} \end{aligned}\right\} \quad \boxed{I_{S-} + I_{S+} = I_D \cdot \left(\frac{1}{t_C} + \frac{r_C}{t_C}\right) = I_D \cdot \left(\frac{1+r_C}{1-r_C}\right) = n_S \cdot v_{th}}$$

In the ballistic device, the current is independent of the channel length.

The question now is how to evaluate the reflection coefficient at the drain ?
Typical values: r_C~0.4 – 0.2

Scattering and fluxes

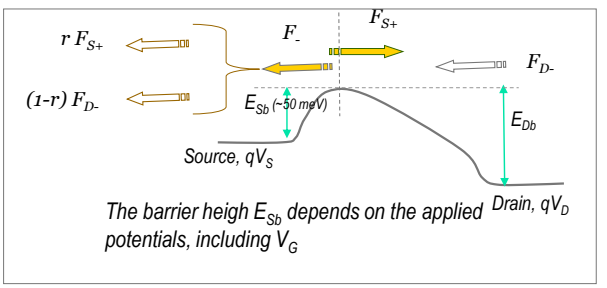
In contrast with former approach, injection from the drain F_{D-} can no longer be neglected if the device operates in linear regime. i.e $V_{DS}<<U_T$

*Under ballistic regime, the fluxes **at the top of the source barrier** can be decomposed into:*

-A source injected flux F_{S+} in the positive direction due to thermal emission over the source barrier.
This occurs for carriers whose longitudinal kinetic energy is greater than the barrier height

- A global negative flux F_{-} that is itself composed of 2 components (can we use same r_C @ S & D):

- *a fraction of F_+ that is backscattered: $r_C F_{S+}$*
- *a fraction of carriers injected from the drain that is transmitted: $(1-r_C) F_{D-}$*



The barrier heigh E_{Sb} depends on the applied potentials, including V_G

A simple relation then exists between F_{S+} and F_{D-} *(non degenerate)* :

$$\frac{F_{S+}}{F_{D-}} = e^{\frac{V_{DS}}{U_T}}$$

When $V_{DS}> 2U_T$, F_{D-} can be neglected.

Scattering and fluxes

Then, the total negative flux at the source barrier is: $F_{-} = (1 - r_c) \cdot F_{D-} + r_c \cdot F_{S+}$

The drain current is then obtained from: $I = W \cdot q \cdot (F_{S+} - F_{-})$

As for the saturation case, we can link the electron density at the source barrier to the global flux of carriers through the thermal velocity:

$$n_S = \frac{F_{S+} + F_{-}}{v_{th}} = \frac{(1 + r_c) \cdot F_{S+} + (1 - r_c) \cdot F_{D-}}{v_{th}}$$

The drain current be expressed in terms of the carrier density @ source barrier and related fluxes:

$$I = W \cdot q \cdot \frac{(F_{S+} - F_{-})}{n_S} \cdot n_S = W \cdot q \cdot n_S \cdot v_{th} \cdot \frac{1 - F_{-}/F_{S+}}{1 + F_{-}/F_{S+}}$$

This relation is almost similar to the saturated case where r_C was simply F_{-}/F_{S+}.

A potential based relation is then recovered.

In very thin silicon layers, calculation of the charge density n_S will have to include discrete energy levels in silicon.
However, only geometrical confinement is important, thus simplifying the model.

$$I = W \cdot q \cdot n_S(V_{GS}) \cdot \left(\frac{1 - r_c}{1 + r_c}\right) \cdot v_{th} \cdot \left(\frac{1 - e^{\frac{-V_{DS}}{U_T}}}{1 + \left(\frac{1 - r_c}{1 + r_c}\right) \cdot e^{\frac{-V_{DS}}{U_T}}} \right)$$

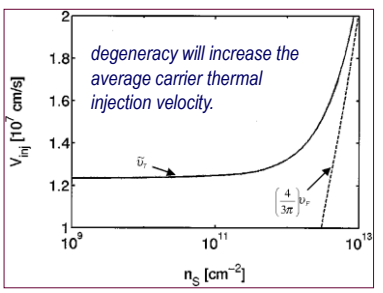
The current is directly affected by back-scattering

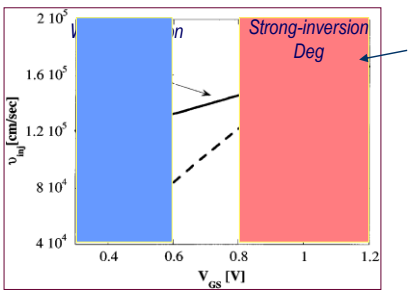
Scattering and fluxes

- ✓ In saturation, the simplified model is recovered: $I \overset{saturation}{\approx} W \cdot q \cdot n_S(V_G - V_T) \cdot \left(\frac{1-r_c}{1+r_c}\right) \cdot v_{th}$
- ✓ *In degenerate and saturation we obtain:* $I \overset{deg-sat}{=} W \cdot \frac{2 \cdot q \cdot \hbar}{3 \cdot \pi^2 \cdot m_t} \cdot \left(\frac{2 \cdot \pi \cdot C_{ox} \cdot [V_G - V_T]}{q}\right)^{3/2}$

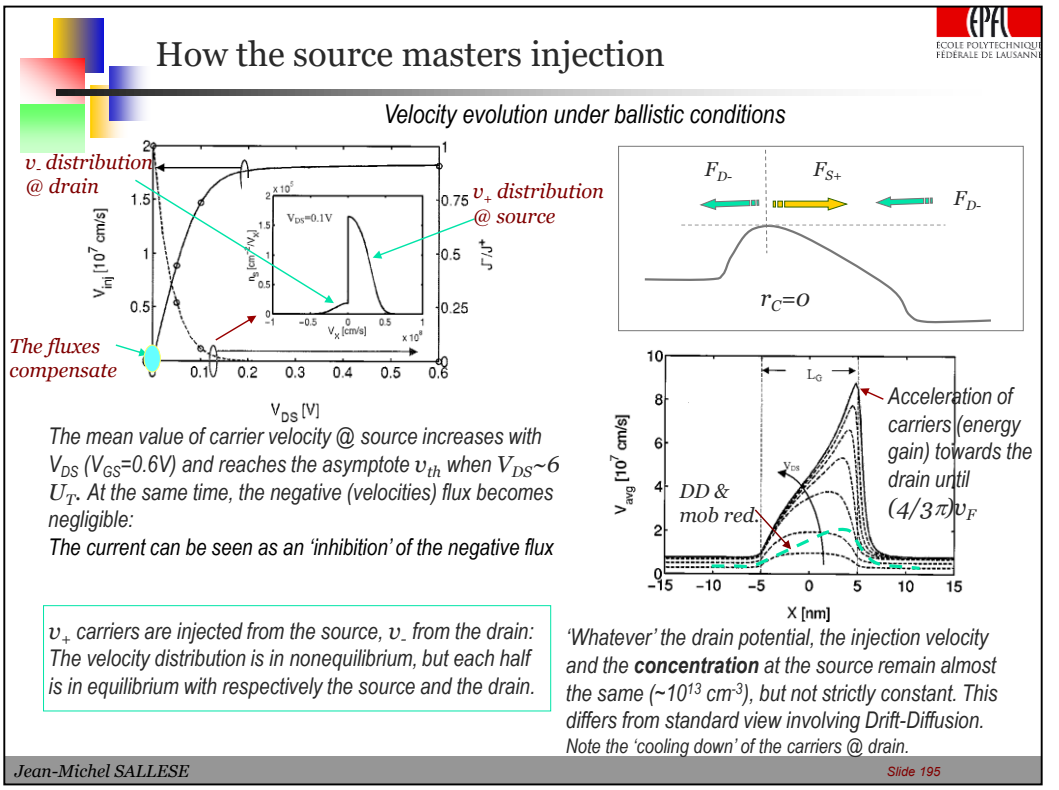
In strong inversion, the degenerate injection velocity is independent of the temperature for a given electron density, i.e. gate voltage. This contrasts with the non degenerate-low inversion operation.

The injection velocity will then depend on V_G.
At high T, electrons start to spread in higher subbands where their velocity will be lower (@ bottom of bands)





V_{inj} is 'always' increased in SI



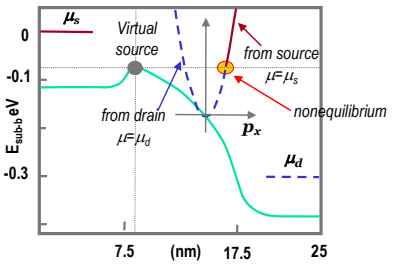
Carriers from the source will populate positive kx states, while thos from the drain kx- states in an hemi Maxwellian distribution.

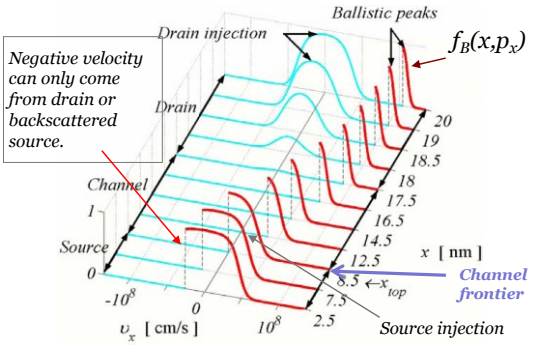
note that the
inversion layer density at the top of the barrier is nearly equal to its equilibrium value in the presence or absence of scattering (this is a simple consequence of self-consistent MOS electrostatics and is relatively insensitive to the specific transport model).

In the channel, carriers gain energy from the channel field.

How the source masters injection

Investigating the electron distribution function (f_B) in the ballistic limit in an ultra-small MOSFET. Shape of the distribution function at different positions under bias





The appropriate Fermi-level to use is that of the contact from which the state was occupied.

Because the Fermi-level change abruptly with energy, highly **nonequilibrium** overall distribution functions are expected. ***No local Fermi*** *energy can be defined* ***in*** *the channel (only at source and drain).*

- Main features of ballistic transport:
  - Asymmetry at the source injecting barrier.
  - Development of the ballistic pics along the channel: carrier that do not scatter are accelerated towards the drain.
  - $[n_{S+}]$ increases with V_{DS} (and $[n_{S-}]$ decreases): hemi-FD instead of Maxwellian.

Jean-Michel SALLESE Slide 196

A semiclassical approach is adequate because
it has been recently demonstrated that the MOSFET's
operate classically down to channel length of about 10
Nm
In equilibriumk when VDS=0, the symmetry of the distribution
is achieved through the balanced injection from each
contact instead of detailed balance due to collision as in
the diffusive regime.

Note, however, that the source injection
increases to maintain the charge balance imposed
by the gate (see the density vs. VDS plot in Fig. 6). Hence,
the area under the positive half of the distribution at
VDS ¼ 0:2 V is approximately twice that of the positive
half of the equilibrium distribution

Thus, the shape of the increasing positive half rapidly
approaches hemi-Fermi–Dirac and its average velocity
vþx
increases up to 1:8 107 cm/s while the diminishing
negative half becomes more hemi-Maxwellian and the average velocity vx approaches the thermal velocity of a hemi-Maxwellian

the ballistic
peak from the source causes heating in the drain region.

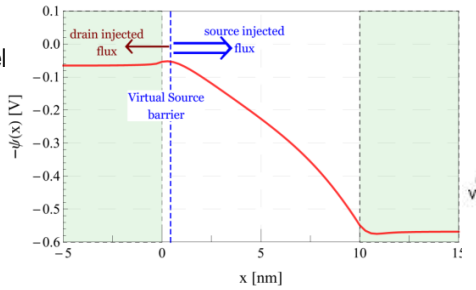
The positive
states are populated by injection from the source, and the negative
states by injection from the drain

Is the ballistic FET a 'Vacuum Tube ' ?

Ballistic FET and 'nano' Vacuum Tube

In the vacuum tube, electrons do not experience any scattering, as in the ballistic FET.
Does it means that both devices behave in the same way ?

The barrier model



The DeForest "Audion" tube

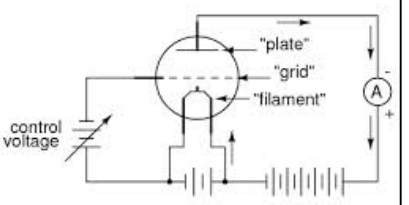
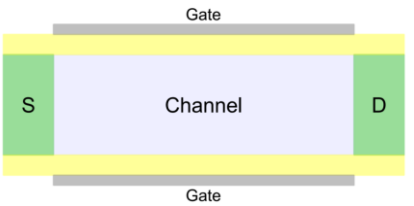


plate current can be controlled by the application of a small control voltage between the grid and filament!



?

A simple model of the Vacuum Tube

Whatever the device, the current density is given by : $I = q \cdot n_x \cdot v_x$

(note that neither drift nor diffusion currents concepts are needed)

In contrast to collision driven current, here carriers will gain energy at the expense of the electric potential : $\frac{1}{2} \cdot m \cdot \left(v_x^2 - v_0^2\right) = q \cdot \left(\psi_x - \psi_0\right)$

This links velocity to the potential once limit conditions are known (Ψ_0 , ρ_0):

$$v_x = \sqrt{\frac{2 \cdot q}{m} \cdot \left(\psi_x - \psi_0\right) + v_0^2} = \sqrt{\frac{2 \cdot q}{m} \cdot \left(\psi_x - \psi_0\right) + \left(\frac{I}{q \cdot \rho_0}\right)^2}$$

At this point, even though the current and the initial conditions are given, the spatial dependence of the potential remains unknown.

But whatever the transport phenomena, Poisson equation should also be satisfied:

$$\frac{d^2\psi_x}{dx^2} = \frac{q \cdot n_x}{\varepsilon} = \frac{q \cdot n_0}{\varepsilon} \cdot \left(1 + \frac{2q^3 n_0^2}{m \cdot I^2} \cdot \left(\psi_x - \psi_0\right)\right)^{-1/2}$$

A simple model of the Vacuum Tube

Making the following substitutions: $\phi_x = \frac{2q^3\rho_0^2}{m \cdot I^2} \cdot (\psi_x - \psi_0)$ $\xi = x \cdot \frac{2q^4\rho_0^3}{\varepsilon \cdot m \cdot I^2}$

Poisson equation constrained to the current continuity gives: $\frac{d^2\phi_x}{d\xi^2} = \frac{1}{\sqrt{\phi_x}}$

Now, limits conditions are given from the potential peak at the virtual source (electric field =0), and given the initial potential:

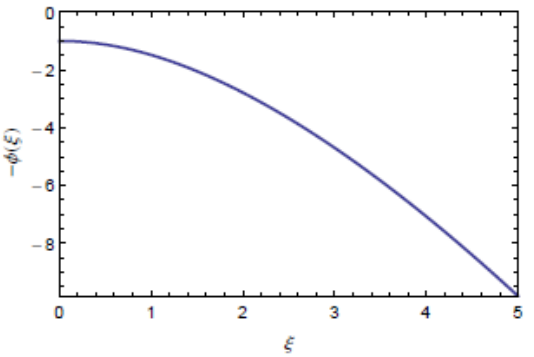
$$\left.\frac{d}{dx}\phi_x\right|_0 = 0 \qquad \phi_0 = 1$$

After some manipulations, an explicit expression is obtained between ζ and Φ:

$$\xi = \frac{2}{3} \cdot \left(\phi_x^{3/2} + 3 \cdot \phi_x - 4\right)$$

Potential distribution in the vacuum tube

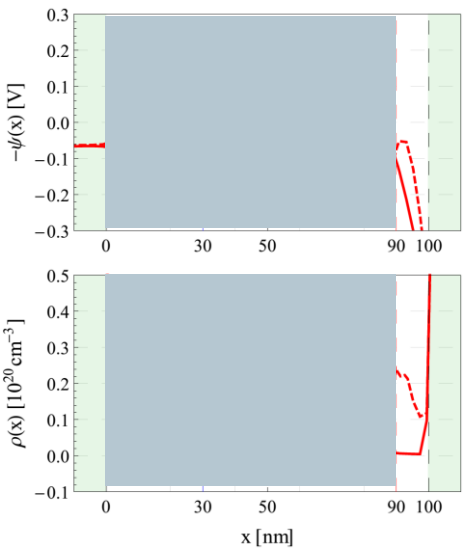
Since ζ and Φ: are proportional to the coordinate and channel potential respectively, the plot depicts the potential distribution inside the channel of the vacuum tube.



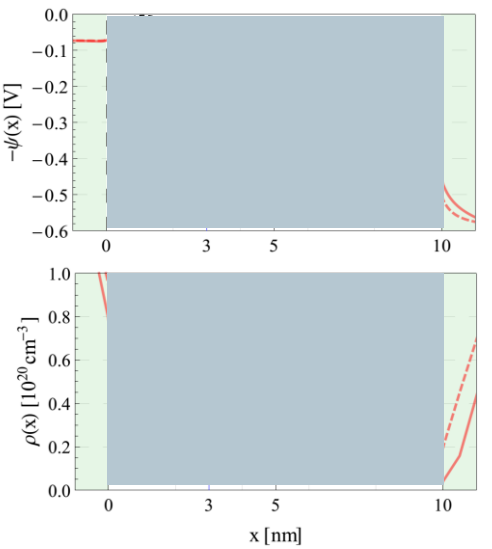
We note that whatever the channel length and the current density, the potential increases continuously between source and drain.

Potential distribution in b-FET (MC simulations)

Channel length $L_c = 100$ nm



Channel length $L_c = 10$ nm



… the Ballistic FET is not always a Vacuum tube…

- In very short channel length, the ballistic FET and the Vacuum tube looks almost the same:

  The drain controls the electrostatics.

- In relatively long channels, ballistic FETs and Vacuum tubes are different devices.

  The gate starts to dominate and controls the electrostatics inside the channel.

  The relationship obtained for the vacuum tube cannot model the potential profile of the b-FET

The ultimate contact resistance

Ballisticity and 'contact resistance'.

At the nano scale limit, we can wonder if the resistance of an infinitely short conductor drops to zero. Following Ohm's law, this should be true since we have $R=\rho L/S$.

Experimentally, it was proven that when the length is reduced below the mean free path, i.e. when transport is ballistic, the conductance reaches a constant value.

To prove this, we evaluate how much current a subband can carry. Each subband has a dispersion relation between the energy and the wave vector k, i.e. E(k).
From band theory, the group velocity of an electron on such level is given by:

$$v(k)=\frac{1}{\hbar}\cdot Grad_k\left(E(k)\right)$$

In a 1D system, the current is simply the sum of all individual electron currents:

$$I=-\frac{q}{L}\cdot\sum_k f(k)\cdot v(k)=-\frac{q}{\hbar}\cdot\frac{1}{L}\cdot\sum_k f(k)\cdot\frac{\partial E(k)}{\partial k}$$

q/L : electron density per unit length
$f(k)$: probability that the state k is occupied

Noting that the 1D density of states $N(k)$ in k-space is $L/2\pi$, and replacing Σ by an integral over k *(· 2 for the spin)*:

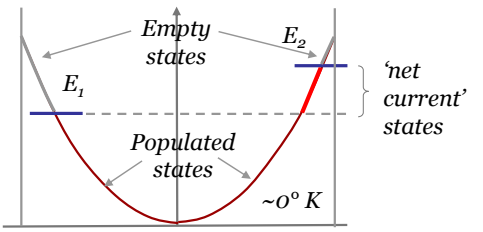
$$I=-2\cdot\frac{q}{\hbar}\cdot\frac{1}{L}\cdot\int_{-\pi/p}^{\pi/p} f(k)\cdot N(k)\cdot\frac{\partial E(k)}{\partial k}\,dk$$

p = period of the potential

The resistance at the interface comes from the difference in the number of modes between the reservoir and the 'channel'. Large reservoir is necessary if the voltage is assumed to be applied on the channel.

Ballisticity and 'contact resistance'.

Now, applying voltage V_1 and V_2, there is a shift in E-k space breaking the symmetry between positive and negative velocities.



At low temperature we can write:

$$I \overset{0K}{=} -\frac{q}{\hbar}\cdot\frac{2}{L}\cdot\int_{k_1}^{k_2}\frac{L}{2\pi}\cdot\frac{\partial E(k)}{\partial k}dk = -\frac{2\cdot q}{h}\cdot\int_{E_1}^{E_2}dE(k) = -\frac{2\cdot q^2}{h}\cdot\frac{E_2-E_1}{q} = \frac{2\cdot q^2}{h}\cdot\Delta V$$

Then, the maximum conductance per mode is $2q^2/h$, independently of the $E(k)$ relation.

The contact resistance of a single mode conductor is then: $R_C = \frac{h}{2\cdot q^2}$

...does not depend on the effective mass !

In a given system, there is not only one mode.
For example, in a quantum well, each confined level will give rise to a conduction mode.
The number of modes M (assumed constant in energy) depends on the device geometry.
These 'parallel' channels will then add up:

Length independent contact resistance

$$R_C = \frac{h}{2\cdot q^2}\cdot\frac{1}{M} = 12.9k\Omega\cdot\frac{1}{M}$$

Ballisticity and 'contact resistance'.

For a conductor of width W, the number of 'channels' (states) is given by:

$$M(k_F) = INT\left(2 \cdot \frac{W}{2 \cdot \pi} \cdot k_F\right) = INT\left(\frac{W}{\lambda_F / 2}\right)$$

spin; 2 DOS

Where k_F is the Fermi Wavelength must satisfy: $\lambda_F = \frac{2\pi}{k_F} << W$

The conductance per unit width of a point contact in 2D is then: $\frac{G}{W} \approx \frac{e^2}{\pi^2 \cdot \hbar} \cdot k_F$

For nanoscaled devices with thin Si film devices, this resistance is a critical parameter: Even more important in short-channel devices due to the increasing influence of R_{SD}, as channel and access resistances become similar.

2DEG AsGa/AlGaAs QW with E_F-E_c~15 meV and m*=0.067: λ_F~38 nm

$$\Rightarrow \quad M(k_F) = INT(5.1 \cdot 10^7 W)$$

$W \approx 100\ nm \Rightarrow M(k_F) = 5 \Rightarrow R \approx 2.6\ k\Omega$

$W \approx 20\ nm \Rightarrow M(k_F) = 1 \Rightarrow R \approx 13\ k\Omega$

Increasing the carrier concentration increases E_F-E_c, and so decreases λ_F: M will increase

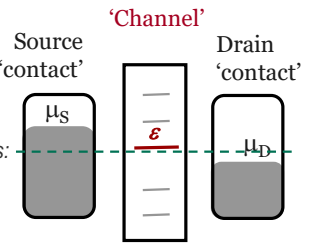
Note that the contact resistance does not scale with W as well, exept for large conductors !

It changes in discrete steps

The ‘molecular’ FET

Simple picture of transport in a 'molecule'

- Transitions operate 'horizontally', i.e. without change in total energy. The mean value for the electron density N (<1) is expected to be an average between $f(\varepsilon-\mu_S)$ and $f(\varepsilon-\mu_D)$.
- *Basically, the current flux generated by the non equilibrium situation between the contacts and the state ε can be written as:* $\underbrace{(1-N)\cdot f(\varepsilon-\mu_S)-(1-f(\varepsilon-\mu_S))\cdot N}$



spin

$$I_{\varepsilon\to S} = q\cdot\frac{1}{\tau_S}\cdot\underbrace{2\cdot\left(f(\varepsilon-\mu_S)-N\right)}=q\cdot\left(\frac{\gamma_S}{\hbar}\right)\cdot 2\cdot\left(f(\varepsilon-\mu_S)-N\right)$$

$$I_{\varepsilon\to D} = q\cdot\frac{1}{\tau_D}\cdot 2\cdot\left(f(\varepsilon-\mu_D)-N\right)=q\cdot\left(\frac{\gamma_D}{\hbar}\right)\cdot 2\cdot\left(f(\varepsilon-\mu_D)-N\right)$$

Attempt frequency of 'escape' from the level ε into source or drain

From the uncertainty principle, we have:

$$\Delta t \approx \hbar/\Delta E$$

Then, γ can be interpreted as an energy broadening, mainly due to the interaction between quantized energies in the contacts and in the channel.

- Under equilibrium conditions, both fluxes must compensate: $I_{\varepsilon\to D} = I_{S\to\varepsilon} = -I_{\varepsilon\to S}$
- This may happen only for a given value of N:

$$N = \frac{\gamma_S}{\gamma_S+\gamma_D}\cdot f(\varepsilon-\mu_S)+\frac{\gamma_D}{\gamma_S+\gamma_D}\cdot f(\varepsilon-\mu_D)$$

N represents an average of the occupation probability as induced by source and drain independently.

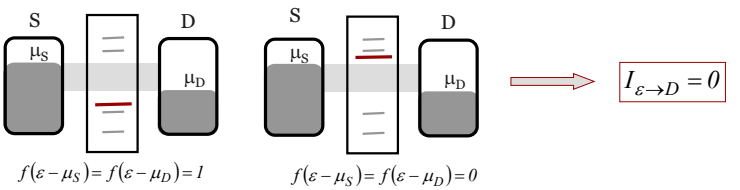
Comment when one of the rates dominates.

Simple picture of transport in a 'molecule'

- Having defined N, the current flowing though the level ε can be calculated *(q>0)*:

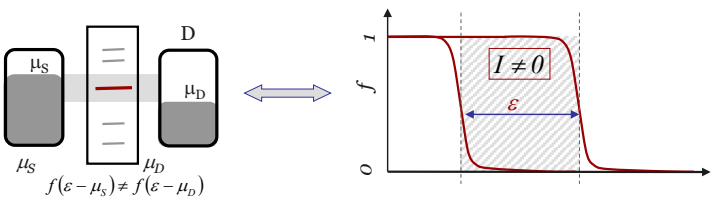
$$I_{\varepsilon\to D} = \frac{2\cdot q}{\hbar}\cdot\frac{\gamma_S\cdot\gamma_D}{\gamma_S+\gamma_D}\cdot\left(f(\varepsilon-\mu_S)-f(\varepsilon-\mu_D)\right)$$

- Can the current always flow?



$f(\varepsilon-\mu_S)=f(\varepsilon-\mu_D)=1$ $\quad$ $f(\varepsilon-\mu_S)=f(\varepsilon-\mu_D)=0$

- A current will then flow only if the level stay between the Fermi energies of the contacts.

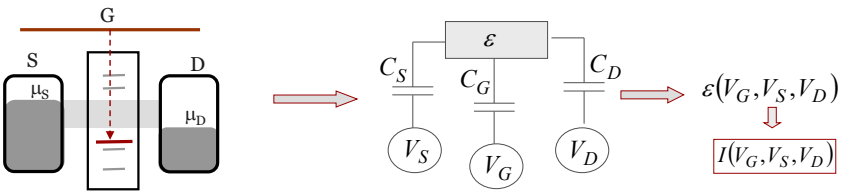


When the level is well below the Fermi energies, we have to rely on Fermi-Dirac statistics, otherwise we will not have fs=fd.

Simple picture of transport in a 'molecule'

The energy level should also depend on the applied potential, which was ignored in the former model. In particular, the level could move mid way between the source and drain potentials. Now, adding an extra gate electrode, we expect that the energy of the level can further be changed independently of the source and drain through capacitive couplings.

- In that case, the gate potential can control the current flow in the 'channel' by pushing the confined state inside or outside the contact energy range.



- The potential of the neutral 'dot' is simply obtained by:

$$C_S\cdot(V-V_S)+C_D\cdot(V-V_D)+C_G\cdot(V-V_G)=0 \Longrightarrow V_0=\frac{C_S}{C_T}\cdot V_S+\frac{C_D}{C_T}\cdot V_D+\frac{C_G}{C_T}\cdot V_G$$

$$C_T=C_S+C_D+C_G$$

- Varying the number of electrons by ΔN, the 'dot' potential must satisfy:

$$C_S\cdot(V-V_S)+C_D\cdot(V-V_D)+C_G\cdot(V-V_G)=-q\Delta N$$

$$V_N=V=\frac{C_S}{C_T}\cdot V_S+\frac{C_D}{C_T}\cdot V_D+\frac{C_G}{C_T}\cdot V_G-\frac{q\Delta N}{C_T}=V_0-\frac{q\Delta N}{C_T}$$

The critical distance is much less than the mean free path

Simple picture of transport in a 'molecule'

- The potential energy is then the sum of two contributions:

$$U_N = -q \cdot V_N = -q \cdot \left(\frac{C_S}{C_T} \cdot V_S + \frac{C_D}{C_T} \cdot V_D + \frac{C_G}{C_T} \cdot V_G \right) + \frac{q^2 \Delta N}{C_T} = -q \cdot V_0 + \frac{q^2 \Delta N}{C_T}$$

Energy raised upon coupling (no charge transfer)

Energy raised upon charging

Note that one could think that the energy could also be calculated from ½ CV². However, this will not represent the potential energy of the dot, but the energy of the electric field in the capacitors.

The energy of the dot will the shifted by U_N...but U_N will also depend on the number of electrons obtained from the steady state current....that will in turn depend on U_N: the system has to be solved self consistently.

$$U_N(\Delta N) = -q \cdot V_0 + \frac{q^2 \Delta N(U_N)}{C_T}$$

In very small dot (~some nm), this may no longer be valid due to the Coulomb blockade effect (confined states will split upon charging).

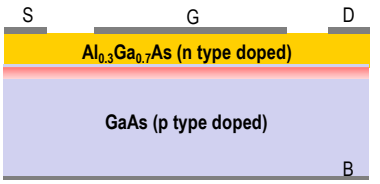
The critical distance is much less than the mean free path

Modeling the High Electron Mobility Transistors (HEMT)

The High Electron Mobility Transistor (HEMT)

Typical structure

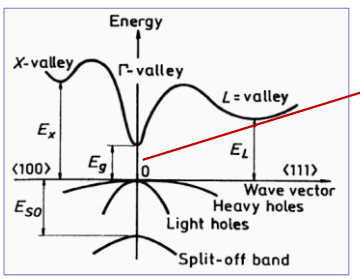
2D electron gas



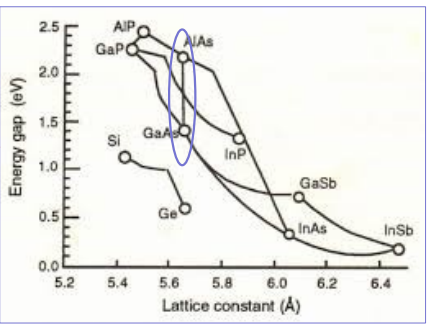
some tens of nm

GaAs substrate or epitaxy (µm range)

Material properties of GaAs and AlGaAs



http://www.ioffe.ru/SVA/NSM/Semicond/AlGaAs/bandstr.html

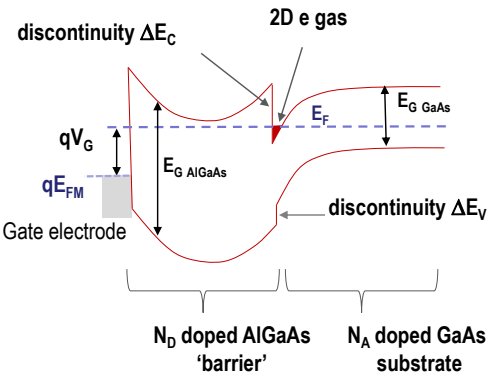


	AlAs	GaAs
Lattice (Amström)	5.66	5.65
Direct band gap (eV)	1.80	1.42

AlGaAs HEMT band structure

The High Electron Mobility Transistor (HEMT)

The abrupt change in the band gap between $Al_{0.3}Ga_{0.7}As$ and GaAs is 'responsible' for a conduction and valence band discontinuity at the heterostructure interface.



Direct band gap $Al_{0.3}Ga_{0.7}As$ = 1.8 eV

Direct Band gap GaAs = 1.42 eV

ΔE_C $Al_{0.3}Ga_{0.7}As$ / GaAs = 0.22 eV

ΔE_V $Al_{0.3}Ga_{0.7}As$ / GaAs = 0.16 eV

Creation of a quantum well

2D electron gas : the channel

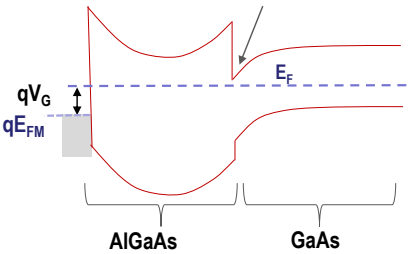
Note that in practice, for the AlGaAs system, the large band gap layer is about 30% Al content. Higher values could create higher band gap discontinuities, but these are not useful because of intrinsic deep levels (DX centers) which 'recapture' free electrons.

The High Electron Mobility Transistor (HEMT)

The 2D conductive quantum well can be modulated with the gate potential.

The 2D e gas fades out

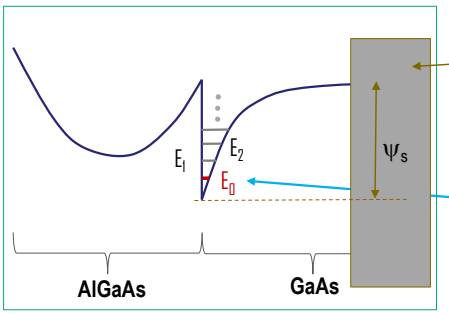
Decreasing the gate voltage will pull up the bands of the AlGaAs layer and pull up the quantum well



Similarities with standard MOSFETs … BUT:

- *No oxide / insulating layer*
- *Use of a Schottky contact between the gate and the wide bang-gap semiconductor (n-type)*
- ***The effective mass of GaAs is only 0.067 m_0…very light electrons : Strong quantization***
- ***2D quantum well needs 2D density of states.***
- ***The Fermi level can cross the confined 'levels' : Fermi Dirac Statistics (deg. semiconductor)***

Modeling Strategy



Neutral beyond this limit, fully depleted otherwise

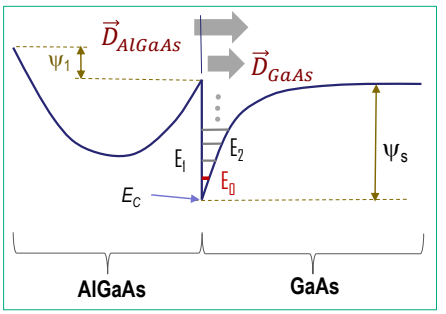
The potential drop in the GaAs, surface potential

Ground state E_0 populated with n_{ch}

Assumptions:

- The AlGaAs barrier layer is lightly doped and totally depleted
- The GaAs bulk is fully depleted in a limited volume (i.e. substrate depletion in MOSFETs)
- The discontinuity in CB creates a 'triangular' quantum well at the heterojunction
- CB in GaAs splits in discrete 2D levels E_i
- E_i depend on the electric field at the heterojunction, i.e. on [electron] in the well.
- The electron density in the quantum well depend on E_i , i.e. on the electric field.

Modeling Strategy: Electrostatics



- *Potential drops in the barrier and in the substrate:*

$$\psi_1 + \psi_S = V_{GB} - \phi_B - \frac{\Delta E_C}{q}$$

- The discontinuity in the displacement vector gives the surface charge density in the channel

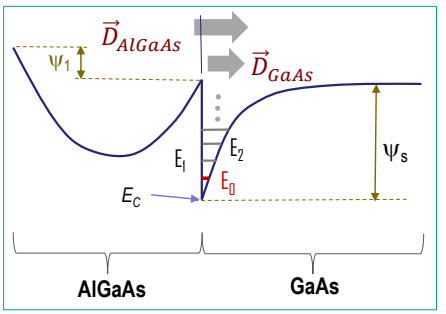
$$D_{GaAs} - D_{AlGaAs} = \varepsilon_{GaAs} E_{GaAs} - \varepsilon_{AlGaAs} E_{AlGaAs} = -qn_{ch}$$

- The charge neutrality :

$$n_G - t_{AlGaAs}\, N_D - t_{Dep} N_A - n_{ch} = 0$$

Charge on the gate — Charge in the quantum well

Modeling Strategy: Semiconductor physics



- The quantized levels depend on the electric field at the interface. This will depend both on the channel density and on the depletion charge in GaAs

- Charge density in the QW (2DOS + Fermi-Dirac)

$$-n_{ch} = 4\pi \underbrace{\frac{m^*}{\hbar^2}}_{\text{2DOS}} U_T \, ln\left(1 + exp\frac{E_F - E_0}{kT}\right)$$

- ...but also (triangular QW)

$$E_0 - E_C = K\, n_{ch}^{3/2}$$

(the depletion charge in GaAs is ignored for simplicity)

$$E_n \approx \left(\frac{\hbar^2}{2 \cdot m}\right)^{\frac{1}{3}} \cdot \left(\frac{3}{2} \cdot \pi \cdot \frac{Q_{Si}}{\varepsilon_{Si}} \cdot \left(n + \frac{3}{4}\right)\right)^{\frac{2}{3}}$$

The concept of charge linearization in HEMT

The concept of charge linearization in HEMT

- Assuming full depletion approximation, the potential drop in GaAs is : $\psi_S = \frac{qN_A}{2\varepsilon_{AlGaAs}} x_{dep}^2$

The barrier capacitance is defined as $C_{AlGaAs} = \left(\frac{\varepsilon_{AlGaAs}}{t_{AlGaAs}}\right)$

The charge neutrality becomes $\frac{Q_{ch}}{C_{AlGaAs}} = -\frac{qn_{ch}}{C_{AlGaAs}} = V_{GB} - V_A - \psi_S - \frac{\sqrt{2q\varepsilon_{AlGaAs}N_A}}{C_{AlGaAs}}\sqrt{\psi_S}$

V_A : *technological parameter* — γ

$V_A = -\frac{\Delta E_C}{q} + \phi_B - \frac{qN_D}{2\varepsilon_{GaAs}} t_{AlGaAs}^2$ *plays the role of the flat band voltage for MOSFETs.*

We obtain the same relationship as for the bulk MOSFET.
The reason is that we are focusing on electrostatics …

$$\frac{-Q_{ch}}{C_{AlGaAs}} = V_{GB} - V_A - \psi_S - \gamma\sqrt{\psi_S}$$

Note that the depleted AlGaAs barrier acts as the gate oxide in MOSFET .

The High Electron Mobility Transistor (HEMT)

$$\frac{-Q_{ch}}{C_{AlGaAs}} = V_{GB} - V_A - \psi_S - \gamma\sqrt{\psi_S}$$

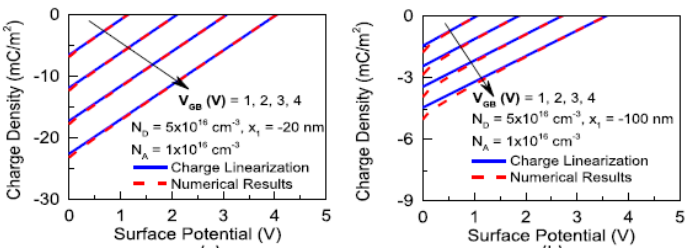
→ **Linearization wrt ψ_S**

$$\frac{Q_{ch}}{C_{AlGaAs}} = n_q(\psi_S - \psi_P)$$

As for the Si-bulk MOSFET, we can define a pinch-off surface potential ψ_P where the mobile charge density cancels, and the slope factor n_q

$$\psi_P = V_{GB} - V_A - \gamma^2\left(\sqrt{\frac{V_{GB} - V_A}{\gamma^2} + \frac{1}{4}} - \frac{1}{2}\right)$$

$$n_q = 1 + \frac{\gamma}{2\sqrt{\psi_P}}$$



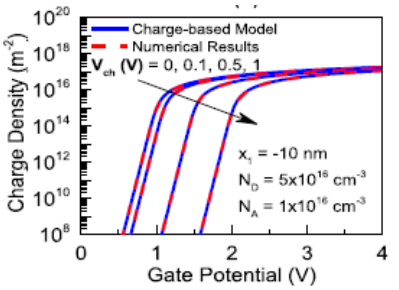
The Charge-Voltage core relationship

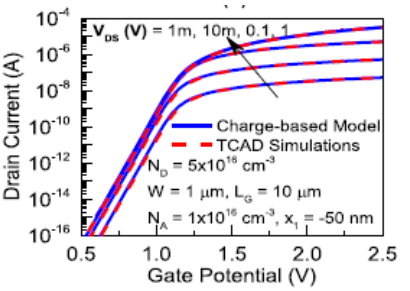
$$U_T \ln\left(\exp\left(\frac{n_{ch}}{U_T\, DOS}\right)-1\right)+\frac{q n_{ch}}{n_q\, C_{AlGaAs}}+\frac{K}{q}\, n_{ch}^{2/3}=\psi_P - V$$

The specific current I_S and inversion coefficient IC $\quad I = q n_{ch} W\, dV_{ch}/dy$

$$I_{Spec} = 2 n_q\, C_{AlGaAs}\, \mu\, U_T^2 \frac{W}{L} \qquad Q_{Spec} = -2 n_q\, C_{AlGaAs}\; U_T \qquad q = Q/Q_{Spec}$$

$$i = I/I_{Spec} = q_s^2 + q_s \; - q_D^2 - q_D$$





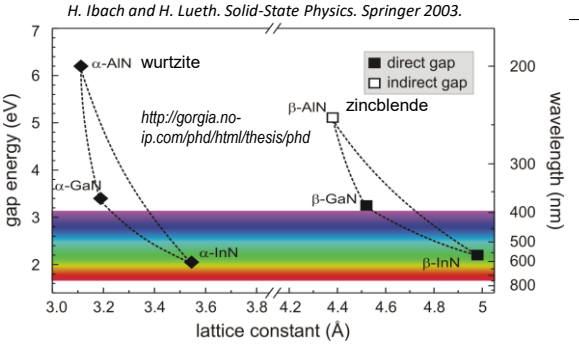
The case of III-Nitride HEMT
AlGaN

The AlGaN - HEMT

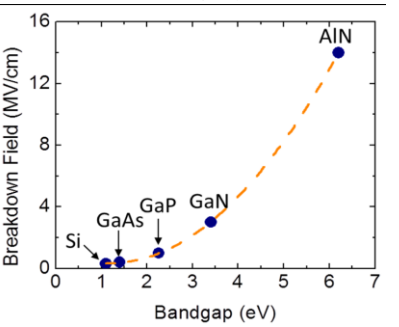
The wide band-gap semiconductors AlGaN HEMTs.

AlGaN semiconductors are large band gap and are used in power applications and optoelectronics.

H. Ibach and H. Lueth. Solid-State Physics. Springer 2003.



Razzak, IMWS-AMP 2018), July 16-18, 2018



In AlGaN a **piezoelectric (due to interface strain) and spontaneous (indpt of the strain) contributions** must be included as well (depend on barrier composition)., i.e P_{GaN} and P_{AlGaN}

The discontinuity in the displacement vector at the surface writes now

$$D_{GaN} - D_{AlGaN} = \varepsilon_{GaN}E_{GaN} - \varepsilon_{AlGaN}E_{AlGaN} + P_{GaN} - P_{AlGaN} = -qn_{ch}$$

The AlGaN - HEMT

These spontaneous and piezoelectric polarizations will modify the channel charge density.
For the same electric field discontinuity at the interface, the channel charge density will be affected

A charge density may exist at 'flat band', i.e. without any electric field …

Polarisation can be accounted in the effective barrier parameter. $\phi_{B\_Eff}=\phi_B+\frac{t_{AlGaN}}{\varepsilon_{AlGaN}}(P_{AlGaN}-P_{GaN})$

This mean that polarization will be merely a shift in V_A, and so a **shift in the 'threshold voltage**

$$V_A=-\frac{\Delta E_C}{q}+\phi_{B\_Eff}-\frac{qN_D}{2\varepsilon_{GaAs}}t_{AlGaAs}^2$$

Note that $P_{GaN}-P_{AlGaN}$ has no contribution to the total space charge density

Modeling AlGaN based HEMT is very similar to AlGaAs Hemts, except that AlGaN-GaN interface is responsible for a piezoelectric-induced spontaneous polarization

This **self-polarization** is responsible for a 'built-in' channel.
It can be taken into account through a gate voltage shift in the model developped for AlGaAs

The AlGaN - HEMT

Some Parameters

Symbol	GaAs	GaN	$Al_xGa_{1-x}As$	$Al_xGa_{1-x}N$
N_C [cm^{-3}]	4.7×10^{17} [22]	1.2×10^{18} [22]	$x < 0.45 \rightarrow 2.5 \times 10^{19}(0.063 + 0.083x)^{3/2}$ $x > 0.45 \rightarrow 2.5 \times 10^{19}(0.85 - 0.14x)^{3/2}$	$(1-x)N_C$ (GaN) $+xN_C$ (AlN)
N_V [cm^{-3}]	8.97×10^{18} [23]	2.28×10^{19} [23]	$2.5 \times 10^{19}(0.51 + 0.25x)^{3/2}$	$(1-x)N_V$(GaN) $+xN_V$(AlN)
$\varepsilon_{1,2}$ [-]	13.1 [23]	9.5 [23]	$13.2 - 2.9x$	$8.9 - 0.4x$
E_g [eV]	1.422 [23]	3.435 [23]	$x < 0.45 \rightarrow 1.42 + 1.25x$ $x > 0.45 \rightarrow 1.9 + 0.125x + 0.143x^2$	E_g (AlN) $x + [E_g$ (GaN) $-1.3x](1-x)$
P_{sp} [C/m^2]	–	0.029	–	$0.029 - 0.052x$
P_{pz} [C/m^2]	–	0	–	$2(e_{31} - c_{13}e_{33}/c_{33})(a_{GaN}/a_{AlN} - 1)x$
ΔE_C [eV]	–	–	$x < 0.41 \rightarrow 0.79x$ $x < 0.41 \rightarrow 0.475 - 0.335x + 0.143x2$	$0.7[E_g$ (AlGaN) $- E_g$ (GaN) $]$
m_e/m_0 [-]	0.067 [22]	0.2 [22]	0.059 [24]	0.228 [25]

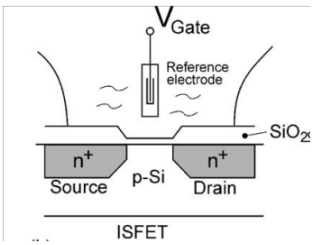
Jazaeri and Sallese - IEEE TRANSACTIONS ON ELECTRON DEVICES, VOL. 66, NO. 3,, pp. 1218 - 1229 (2019)

FETs Bio-Sensors

Jean-Michel Sallese EPFL

Generality on FET bio-sensors

- Ion Sensitive FETs exist for a long time.
  - Bergveld in the 1970's



Cross section of an ISFET

potential at the insulator-electrolyte interface

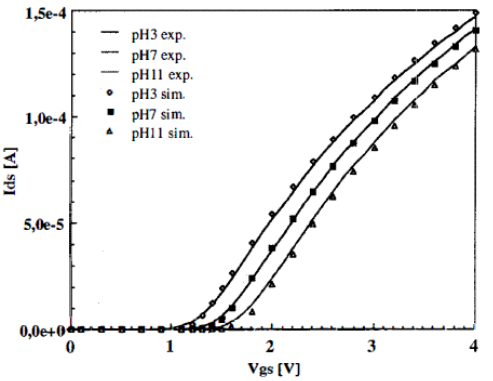
$$\Delta\varphi_s = K\,\Delta pH_{sol}$$

- Principle remains the same as for 'regular' FETs:
  - The gate electrode is 'removed'.
  - The solution acts as a 'liquid' gate
  - **The potential at the SiO_2-liquid interface may also depend on ions present in the solution : this is used to sense the pH**

Bergveld, IEEE Trans. on Bio-medical Engin, p. 70 (1970).

Generality on FET bio-sensors

- Typical I-V characteristics of a MOSFET when varying the pH



A Lui et.al, Microelectronic Test Structures conf, 1996

- Modeling this biosensor requires :
  - a **double layer model** that will describe the potential distribution between the electrolyte and the solid.
  - a model that describes the **adsorption of protons**, the **site binding model**.

Basics of solid-electrolyte interaction

The Site Binding Model: charge at the surface

The surface of SiO_2 gate dielectric is terminated by hydroxyl groups , OH. (i.e. silanol Si-OH)
These may act as proton donors or acceptors, depending on the electrolyte pH: Amphoteric.

Three different types of sites: neutral (A-OH), negative ($A\text{-}0^-$) or positive ($A\text{-}H_2O^+$).

If N_s is the total number of sites (per unit surface) on the oxide surface, we have:

$N_S = [A\text{-}H_2O^+] + [A\text{-}O^-] + [A\text{-}OH]$

The charge distribution will depend on the pH of the solution at the electrolyte-solid interface

Low pH : $A\text{-}OH \Longleftrightarrow A\text{-}0^- + H^+$ $\quad K_a = [A\text{-}0^-]\ [H^+] /\ [A\text{-}OH]$

High pH : $A\text{-}OH + H^+ \Longleftrightarrow A\text{-}H_2O^+$ $\quad K_b = [A\text{-}H_2O^+]\ /\ [A\text{-}OH]\ [H^+]$

The pH at the surface is a function of the reaction constants and charges at the surface :

$[H^+]^2 = K_a\,[A\text{-}H_2O^+] / K_b\,[A\text{-}0^-] \Longrightarrow pH_{surf} = -\,\text{Log}\,[H^+] = 0.5\,\text{Log}\,([A\text{-}0^-]\,K_b / [A\text{-}H_2O^+]\,K_a)$

Remember $pH = -log_{10}[H^+]$

The Site Binding Model: charge at the surface

The net charge density at the surface is given by (q>0) $\sigma = -q\,([A\text{-}O^-] - [A\text{-}H_2O^+])$

So, σ depends on the concentration of protons $[H^+]$ at the surface.

$$[A-H_2O^+] = N_s \frac{K_b[H^+]^2}{K_a + [H^+] + K_b[H^+]^2}$$

$$[A-O^-] = N_s \frac{K_a}{K_a + [H^+] + K_b[H^+]^2}$$

$$\Rightarrow \quad \sigma = -q\,N_s \frac{K_a - K_b[H^+]^2}{K_a + [H^+] + K_b[H^+]^2}$$

The surface stores charges as result of a change in H^+ concentration **at the surface**

The 'Point of Zero Charge' pH : pH_{pzc}

When the surface is neutral, the pH at the surface is known as the Point of Zero Charge pH :

$[A\text{-}H_2O^+] = [A\text{-}O^-] \Rightarrow [H^+]^2 = K_a / K_b \Rightarrow pH_{pzc} = -\log [H^+] = 0.5 \log(K_b / K_a)$

pH_{pzc} is a characteristic of the oxide . It doesn't depend on the surface sites density N_s

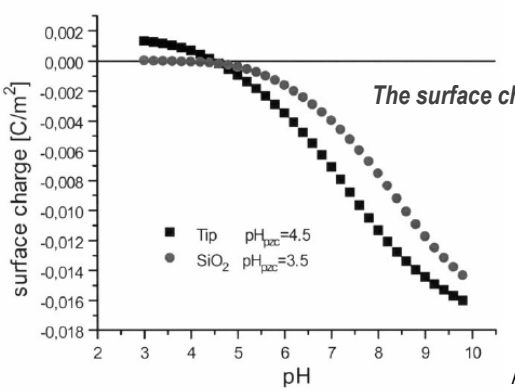
The Site Binding Model: charge at the surface

Rough estimation of the surface charge

At high pH (low [H⁺]) : $\sigma \cong -q\,N_s$

At low pH (high [H⁺]) : $\sigma \cong q\,N_s$

The variation of the surface charge it $2qN_S$ at the most ...



The surface charge can change sign

Raiteri et. al. Sensors and Actuators B 46 (1998) 126–132

Modeling electrolyte-solid interface potential

Implications of the surface charge density:

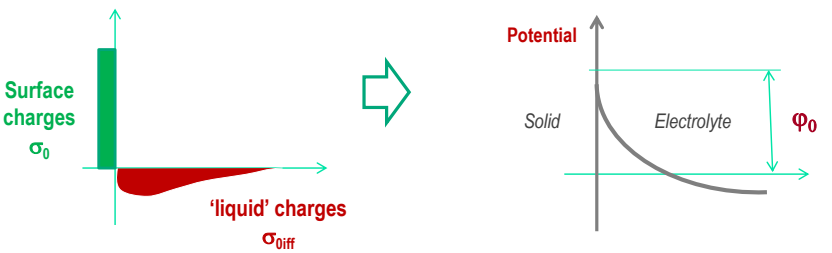
The charge stored on the solid surface will generate a potential in the liquid

The charge stored on the insulator-liquid interface must be compensated by a charge in the liquid…or ***somewhere else …***

The electrolyte contains anions and cathions

The charge in the electrolyte is accompanied with a variation in the electrostatic potential $\psi(x)$.

We can use Boltzman statistics to link the local charge density with the local potential .



Modeling electrolyte-solid interface potential

Initially, **Helmholtz** proposed that the opposite charge in the electrolyte is a sheet layer, a monolayer of opposite polarity, separated by a distance of molecular order..

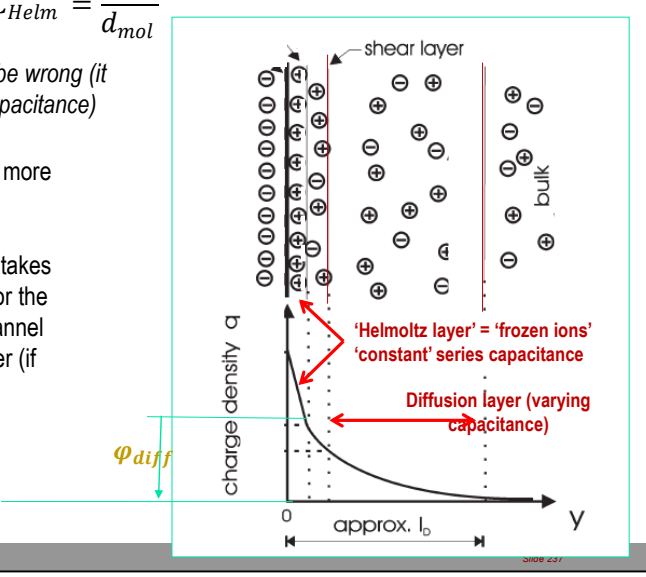
The capacitance would then be $C_{Helm} = \frac{\varepsilon}{d_{mol}}$

Experimentally, this was proven to be wrong (it predicts a potential independent capacitance)

Later **Gouy-Chapman** proposed a more elaborated theory.

The rearrangement of the charges takes place in a finite thickness, just as for the depletion charge in a MOSFET channel that coexists with the inversion layer (if any): **the diffusion layer**

Barz et. al.
Lab Chip, 2005, 5, 949–958

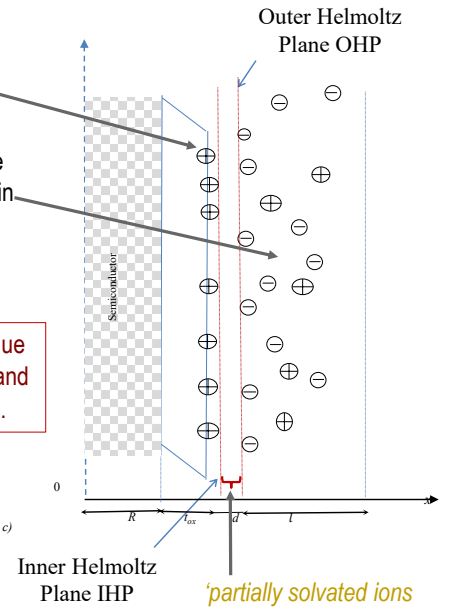


Modeling electrolyte-solid interface potential

ions adsorbed due to chemical interactions (before IHP)

Ions with the same sign wrt the charge on the oxide surface will be repelled, creating **a diffusion layer** in the electrolyte (beyond OHP).

For each surface-electrolyte system, there is a unique correspondence between the pH in the electrolyte and the charge density at the oxide-electrolyte interface.



Modeling electrolyte-solid interface potential

Depending on the nature and concentration of ions present in the solution, this surface charge creates an electrostatic potential φ

The Gouy Chapman model

*In presence of ion species i having a charge z_i, the **Poisson Boltzmann equation** reads: (and the concentration at equilibrium in the solution is $C_{sol,}$)*

$$\frac{d^2\varphi}{dx^2} = \frac{q}{\varepsilon}\sum C_{sol_i}\, z_i\, e^{z_i\frac{\varphi}{U_T}}$$

Assuming that the charge on ions is +/- zq (1:1 electrolyte):

$$\frac{d^2\varphi}{dx^2} = \frac{q}{\varepsilon} C_{sol}\left(e^{z\frac{\varphi}{U_T}} - e^{-z\frac{\varphi}{U_T}}\right)$$

The limit conditions are the same as for bulk MOSFET:
Electric field cancel at infinity, as for the potential (much simpler than in a DG FET) .
Then, in the diffusion layer, the charge stored depends on the potential drop as:

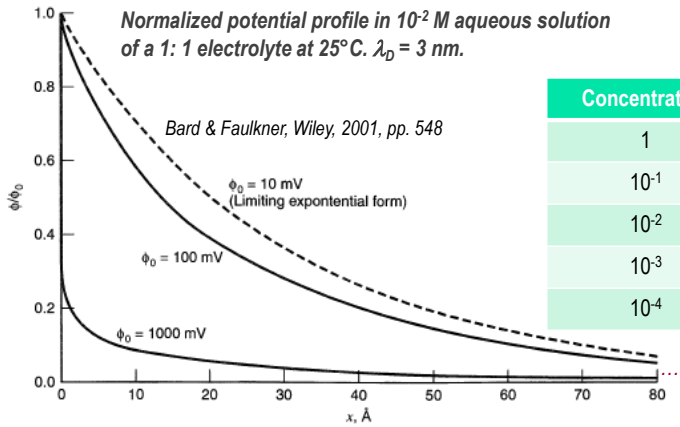
$$\sigma_{diff} = -\sqrt{8qU_T\varepsilon C_{Sol}}\, sinh\left(\frac{z\varphi_{diff}}{2U_T}\right)$$

(note that this kind of assymetry in charge-vs-potential is possible because all the charges are 'mobile…not as in MOSFETs

Modeling electrolyte-solid interface potential

In case of a small potential drop:

$$\varphi = \varphi_0 \, e^{-\frac{x}{\lambda_D}} \qquad \lambda_D = \sqrt{\frac{\epsilon U_T}{2Cz^2q}}$$



Normalized potential profile in 10^{-2} M aqueous solution of a 1: 1 electrolyte at 25°C. λ_D = 3 nm.

Bard & Faulkner, Wiley, 2001, pp. 548

Concentration	λ_D (nm)
1	0.3
10^{-1}	0.96
10^{-2}	3
10^{-3}	9.6
10^{-4}	30

… very small screening distances…

Still, the model is insufficient as it predicts a larger capacitance than what gives experiment. Is some fixed 'Stern' layer missing ?

$$C_D = d\sigma/d\varphi$$

Modeling electrolyte-solid interface potential

The Gouy–Chapman–Stern Model:

A combination of the Helmholtz layer in series with the Gouy–Chapman Model .

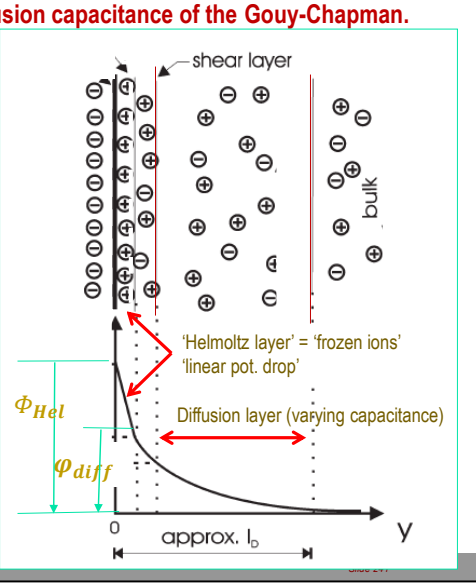
The idea is to add a series capacitance to the diffusion capacitance of the Gouy-Chapman.

$$C_{Total}^{-1} = C_{Helm}^{-1} + C_{Gouy}^{-1}$$

The Helmoltz layer is a kind of 2D dipole with respect to the charge sheet on the solid: the electric field is constant and the potential drops linearly.
The potential in the Gouy –Chapman model is then the potential at the 'exit' of this Helmoltz layer

Beyond this 'outer Helmotz plane', we can create an implicit link between Charges and Potentials.

$$= -\sqrt{8qU_T\varepsilon C_{Sol}}\ sinh\left(\frac{z\left(\varphi_{diff} + \frac{}{C_{Helm}}\right)}{2U_T}\right)$$



pH at surface vs pH in the electrolyte

The electrostatic potential links the concentration of protons between the surface and the liquid, and so the pH at the surface will not be the same as in the 'neutral' electrolyte .

$$[H_{surf}] = [H_{sol}]\, e^{\frac{-\varphi_0}{U_T}} \quad \Rightarrow \quad pH_{surf} = pH_{sol} + \frac{\varphi_0}{2.3U_T}$$

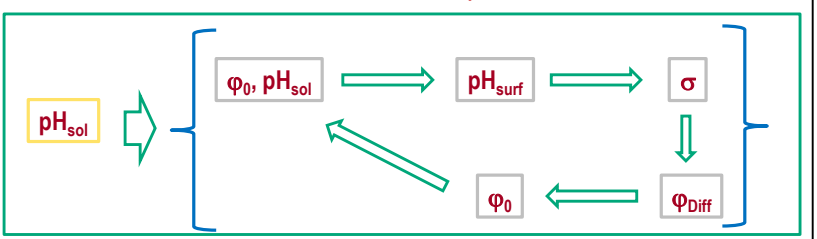
But....

$$\sigma_0 = -\sigma_{\text{diff}} = -q\, N_s \frac{K_a - K_b[H^+]^2}{K_a + [H^+] + K_b[H^+]^2}$$

And ....

$$= -\sqrt{8qU_T\varepsilon C_{Sol}}\, sinh\left(\frac{z\left(\varphi_{diff} + \zeta_{Helm}\right)}{2U_T}\right)$$

The steady state solution will have to be solved self-consistently.



Estimation of the theoretical sensitivity

The sensitivity of the FET will depend on the link between the surface potential at the liquid-solid interface, and the pH of the solution: $d\varphi_0/d\,pH_{sol}$

$$\frac{d\varphi_0}{d\,pH_{sol}} = \frac{d\varphi_0}{d\,\sigma}\,\frac{d\,\sigma}{d\,pH_{surf}}\,\frac{d\,pH_{surf}}{d\,pH_{sol}}$$

$$\frac{d\sigma}{d\,pH_{surf}} = -\beta q$$

β is intrinsic buffer capacity …will be detailed later

$$\frac{d\varphi_0}{d\,pH_{sol}} = \frac{-\beta q}{C_T}\,\frac{d\,pH_{surf}}{d\,pH_{sol}}$$

$$\frac{d\varphi_0}{d\,pH_{sol}} = -U_T\,\alpha \qquad \text{With} \qquad \alpha = \frac{1}{1 + 2.3\,\frac{U_T}{q\beta}\,C_T}$$

0< α <1 *. Depends on the intrinsic buffer capacity of the surface (which is also a function of the pH) and on the capacitance.*

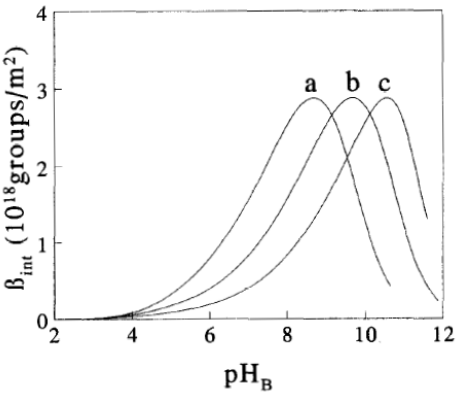
The intrinsic buffer capacity

The ability of the surface to store charge as result of a change in the H+ concentration at the surface is given by the **intrinsic buffer capacity β .**
It links the variation in the number of charged groups, i.e. the variation in σ, to the variation of the pH at the surface :

$$\beta = \frac{d\left[-\frac{\sigma}{q}\right]}{d\ pH_{surf}} = \frac{d([A-O^-]-[A-H_2O^+])}{d\ pH_{surf}}$$

$$-q\beta = \frac{d[\sigma]}{d\ pH_{surf}} = \frac{d[\sigma]}{d[H^+]}\frac{d[H^+]}{d\ pH_{surf}} =$$

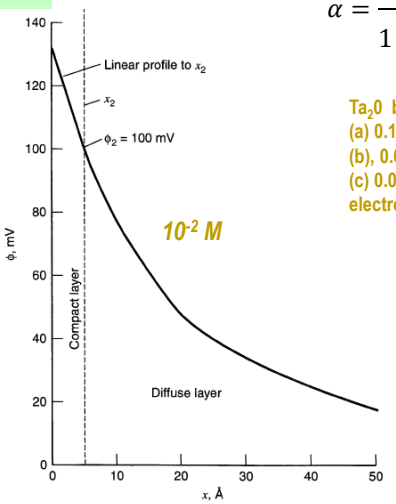
$$-q\ N_s \frac{K_b[H^+]^2 + 4K_aK_b[H^+] \quad + K_aK_b^2}{\left(K_aK_b + K_b[H^+] \quad + [H^+]^2\right)^2} 2.3[H^+]$$



R.E. G. Huan Hal et al.I Adu. Colloid Interface Sci. 68 (1996) 31-62

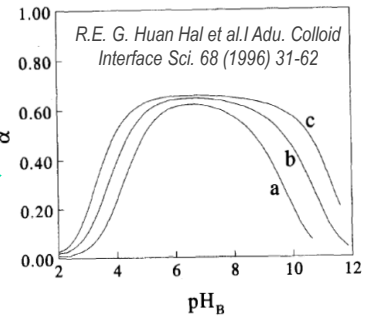
Some orders of magnitude

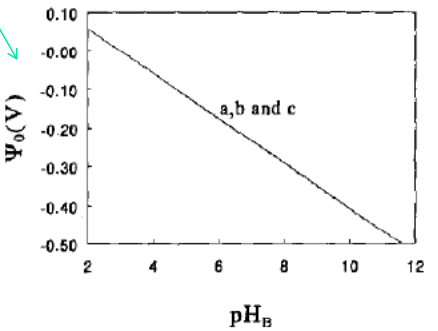
$$\alpha = \frac{1}{1 + 2.3\,\frac{U_T}{q\beta}\,C_T}$$



Bard & Faulkner, Wiley, 2001, pp. 548

Ta_2O based ISFETs vs pH in
(a) 0.1 M,
(b), 0.01 M
(c) 0.001 M
electrolyte





Modeling ISFET Nanowires

Modeling DG JL-ISFET

Likewise a conventional JL FET with gate electrodes replaced by the electrolyte.

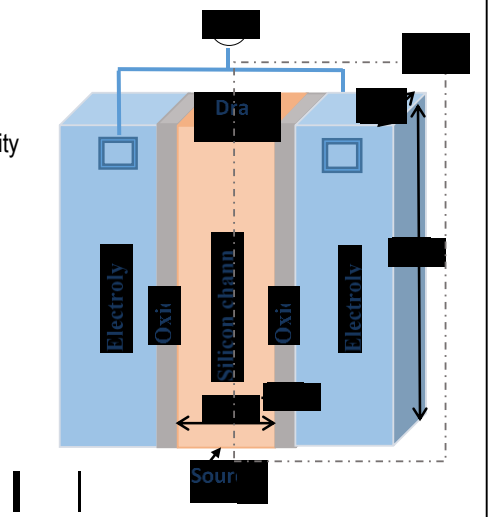
BUT

the effective 'gate voltage' will depend on the physico-chemical properties of the electrolyte and surface affinity to protons (i.e. pH).

The model must **treat on the same ground** the electrostatics of the semiconductor, the electrolyte and the surface chemical reaction.

With respect to the former analysis, we must now also take into account the channel charge density within the charge neutrality condition…

.. And this charge varies along the channel ….



Modeling DG JL-ISFET

The charge on the oxide surface Q_{s_o} depends on the pH (Φ_{ox} is the potential in the electrolyte)

$$ln(10)\left(pH_{zpc} - pH\right) = \frac{\Phi_{ox}}{U_T} + arcsinh\left(\frac{Q_{s_o}}{qN_s\delta}\right)$$

The charge in the Helmholtz layer depends on Φ_{el} , i.e. the potential drop with respect to the neutral electrolyte.

$$Q_{diff\_el} = 2\varepsilon_1\varepsilon_o \frac{U_T}{l_D} \sinh\left(\frac{\Phi_{el}}{2\,U_T}\right)$$

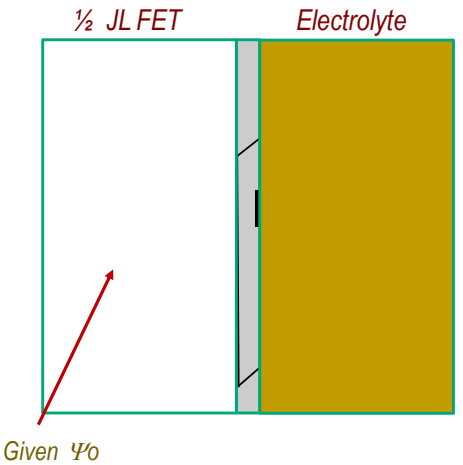
$l_D = (\varepsilon_1\varepsilon_o\, U_T/2qn_o)^{1/2}$ is the Debye screening length of the electrolyte and n_o is the concentration of ions per unit volume in the solution.

Charge neutrality $Q_{sc} + 2(Q_{s\_o} + Q_{diff\_el}) = 0$

Q_{SC} *the total charge density in the semiconductor*

Modeling DG JL-ISFET

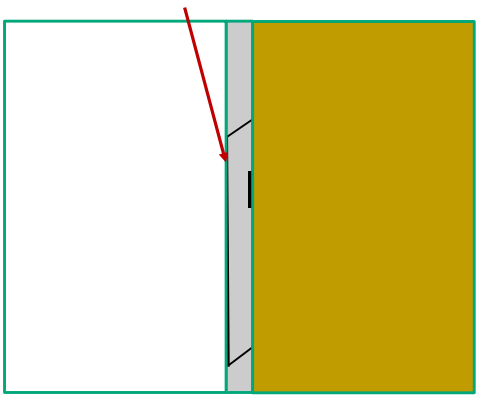
Step 1



Step 2

Link between surface and center potentials (non-full depletion appr.)

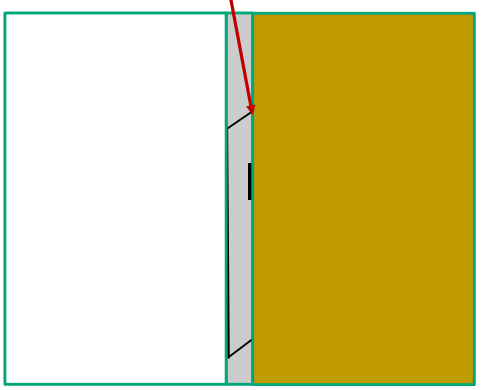
$$\psi_s = \frac{qT_{Si}^2}{8\,\varepsilon_{si}}\left(n_i\,exp\left(\frac{\psi_0 - V}{U_T}\right) - N_D\right) + \psi_0,$$



Modeling DG JL-ISFET

Step 3

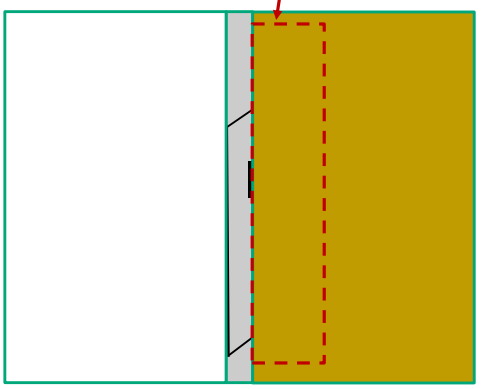
$\psi_s + E_s t_{ox}$



Surface electric field known as well.

Step 4

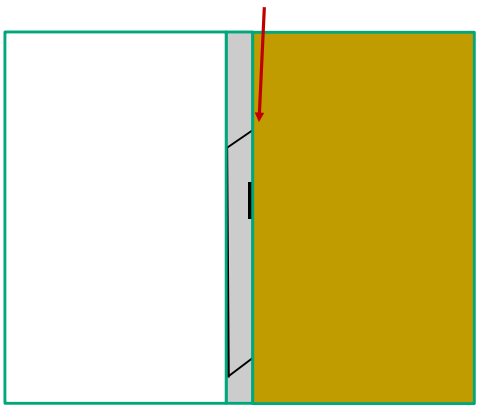
Charge density per unit area in the electrolyte (Q_{s_o} bonded on the oxide surface and (diffusion charge) vs Φ_{ox} and pH in bulk.



Modeling DG JL-ISFET

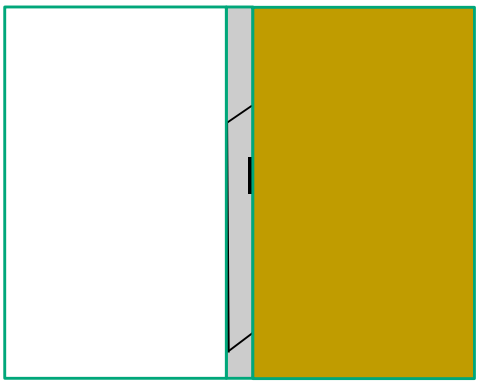
Step 5

Continuity of the electric field at the oxide-electrolyte interface



Step 6

Back to step 4 until charge neutrality is satisfied



If charge neutrality satisfied for the entire device

V_G

Modeling the Nanowire ISFET

A correspondence in terms of equivalent parameters was already proposed for the nanowire JL FET

CORRESPONDENCE BETWEEN NW PHYSICAL PARAMETERS AND EQUIVALENT DG MODEL PARAMETERS

Equiv.DG	$T_{SC}= 2R$	C_{ox}^{NW}	$n_i/2$	$N_D/2$	$W=\pi R$
NW	R	C_{ox}^{NW}	n_i	N_D	

In case of nanowire JL ISFET, the electrolyte is also 'cylindrical'.
Satisfies the Poisson Boltzmann equation in cylindrical coordinates: $\frac{d}{dr}\left(\frac{d\,\Phi}{dr}\right)+\frac{1}{r}\frac{d\,\Phi}{dr}=\eta\,sinh\left(\frac{\Phi}{U_T}\right)$

We use a 'trapezoidal' integration from r=0 to 3 l_d:

$$\int_{R^*}^{R^*+l}\frac{1}{2}\frac{d}{dr}\left(\frac{d\,\Phi}{dr}\right)^2 dr+\int_{R^*}^{R^*+l}\frac{1}{r}\left(\frac{d\,\Phi}{dr}\right)^2 dr=\eta\int_{R^*}^{R^*+l}\frac{d\,\Phi}{dr}\cdot \sinh\left(\Phi/U_T\right) dr$$

We obtain the same relation as for the DG but l_D is now replaced with an *Equivalent Debye length in cylindrical shape.*

$$l_D^*=l_D\sqrt{\frac{R^*+3l_D}{R^*}}$$

Modeling the Nanowire ISFET

As for the semiconductor, the electrolyte in cylindrical geometry can be modelled as a planar one with equivalent parameter:

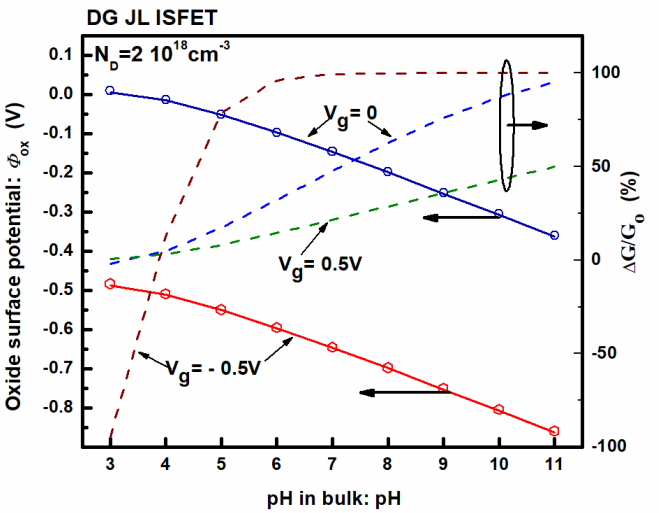
Equivalent parameters of NW JL ISFET in terms of DG JL ISFET

Physical parameters	DG JL IS FET	NW JL IS FET
NW radius	-	R
Semiconductor thickness	T_{sc}	$T_{sc}^* = 2\times R$
Semiconductor width	W	$W^* = \pi\times R$
Doping concentration	N_D	$N_D^* = N_D/2$
Intrensic carrier concentration	n_i	$n_i^* = n_i/2$
Oxide thickness	t_{ox}	$t_{ox}^* = R\,(\ln(1+ t_{ox}/ R)$
Stern layer thickness	d	$d^* = =(R+t_{ox})\cdot\ln[1+d/(R+ t_{ox})]$
Diffuse layer thickness	l_D	$l_D^* = l_D\,[\,(R^*+\nu\, l_D)/ R^*]^{1/2}$ where $R^* = R+t_{ox}+d,\ \nu=3$

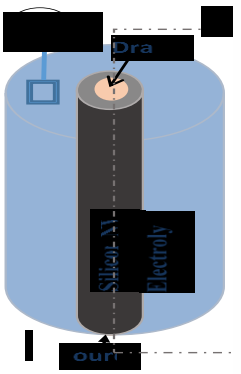
Simulations of ISFET Nanowires

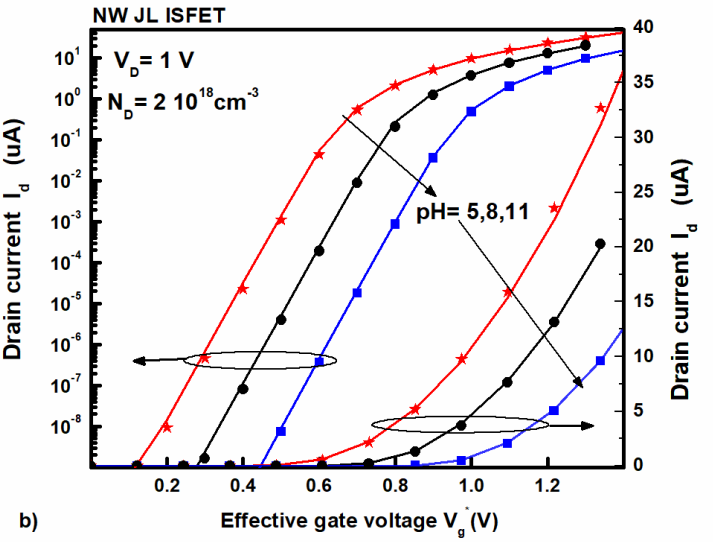
Simulations: DG JL ISFET

Oxide surface potential and relative change in the conductivity upon pH (V_{DS}=10mV) for different gate voltages.



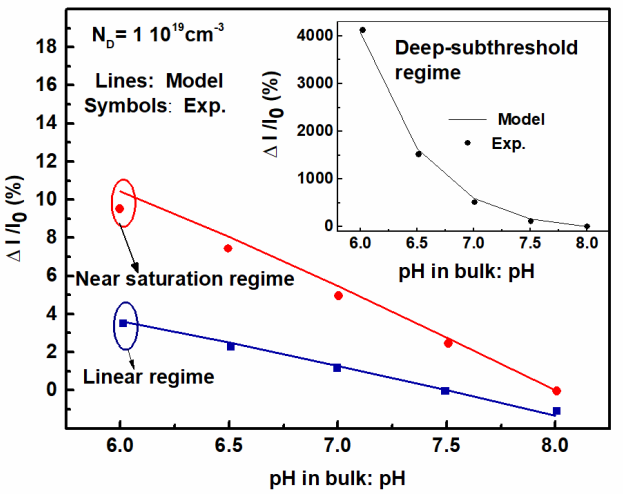
Simulations: Nanowire JL ISFET





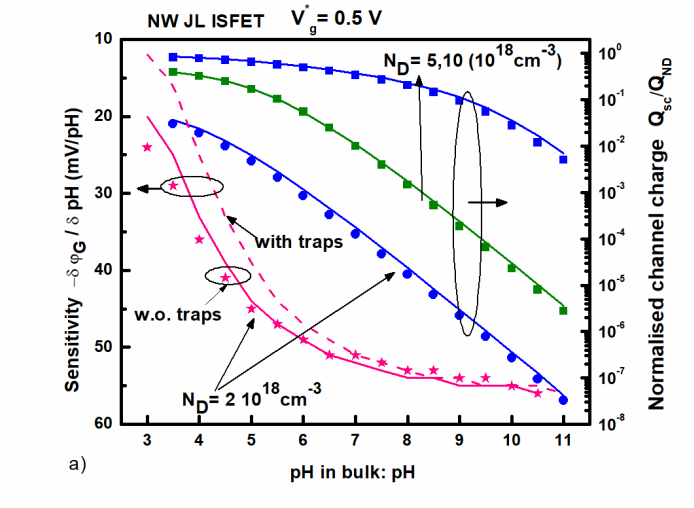
Simulations: DG JL ISFET

Linear regime (V_G=1V) and in nearly saturation regime (V_G=0V), V_{DS}=0.1V



Simulations: DG JL ISFET

Derivative of the oxide surface potential and normalized charge density versus pH



normalized to $Q_d = qN_DT_{Si}$